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CAPABILITY INDICES FOR CONTROL CHARTS BASED ON REGRESSION MODELS

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Abstract

Process capability analysis is extremely important for optimization and quality improvement. It verifies whether the process under analysis is capable of producing items within engineering and customers' specifications. The use of capability indices when assumptions are not satisfied leads to erroneous conclusions, compromising the study and analysis of the process, jeopardizing the fulfillment of requirements from management or external customers. Aiming at filling a gap identified in the literature, the main contributions of this work are: (i) proposition of capability indices for processes monitored through control charts based on regression models, for symmetric and asymmetric specifications; and (ii) comparison of the proposed indices with traditional capability indices through a simulated process.

Palavras-chave: Process control. Regression models. Capability indices.

1. INTRODUCTION

Statistical Process Control (SPC) is an efficient technique that consists in methods of comprehension, monitoring and improvement of process performance over time (WOODALL, 2000). SPC aims at detecting problems and making their identification easier for variability reduction – it is essential for quality and reliability improvement, in addition to the reduction of costs related to low quality of manufactured products. Knowledge about the process – by the means of control charts and capability indices – is essential for granting quality in company.

In the literature, the most widespread control charts are the traditional Shewhart charts, in reason of their simplicity and ease of understanding and formulation. Such charts consider data as independent and identically distributed around a constant mean (COSTA *et al.*, 2005; MONTGOMERY, 2004). These assumptions may not be satisfied when there are frequent changes in the setting of the control variables (independent variables) of the process, because in this case there is a change in data mean and variability, which requires a control chart for each new setting of the machine (PEDRINI and CATEN, 2011; LOREDO *et al.*, 2002). This fact makes SPC operationalization more difficult, as a new Shewart chart would have to be settled for each new setting, in addition to the difficulty of calculating control limits due to the low number of samples of each batch manufactured in each setting. In these cases, the response variable (dependent variable) of a product or process is best represented by a mathematical equation that models its relationship with the control variables of the process (JACOBI *et al.*, 2002; SHU *et al.*, 2004).

Mandel (1969) proposed the regression control chart, a combination of control chart techniques and simple linear regression models. This chart is used in processes in which the effect of a response variable is a function of a control variable. Initially a regression model is generated, which represents the relationship between the response variable and the control variable, and subsequently the residual is monitored through the model. Traditional control charts are not able to perform analyses when the response variable is dependent on the control variable, as a constant average over time is presumed.

Mandel's original proposal (1969) can only be applied in processes involving a control variable. Haworth (1996) extended Mandel's proposal (1969) by suggesting the multiple regression chart, consisting in the estimation of a multiple linear regression model and monitoring of standardized residuals. However, Haworth's work (1996)

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does not present a method clear enough for the chart application. Pedrini *et* Caten (2011) proposed a simple and efficient method for the application of the regression control chart in monitoring of production processes with one or more control variables.

In brief, control charts verify the stability of a process. However, the capability indices assess whether the process is able of meeting both the engineering/project specifications and the customers' specifications. It is seen that a process can be under statistical control, but if its variability is greater than the amplitude of specifications, it will be considered as not able, and corrective actions should be taken in order to reduce the system's variability. Traditional indices, namely the indices applied to control charts proposed by Shewhart, are the most widespread, being their ease of application a highlight. However, there is a gap in the literature: the lack of capability indices specific for processes monitored with control charts based on regression models.

Thus, the objective of this article is to fill this gap, and the main contributions are: *(i)* proposal of capability indices for control charts based on regression models, represented by GR indices, in this study, meeting symmetric and asymmetric specifications; and *(ii)* comparing the capability indices proposed with traditional capability indices by applying a simulated process.

This article is divided into six sections. In addition to this introduction, section two presents a theoretical framework addressing process capability indices and regression control charts. Section three presents the methodological procedures used. In section four, there is the proposal of calculation of capability indices for control charts based on regression models. Section five presents the comparative analysis of the proposed indices and the traditional indices. Finally, section six summarizes the main findings of this study, with suggestions for future studies.

2. THEORETICAL FRAMEWORK

2.1. Capability indices

The main objective to be achieved when studying capability indices is to have them serving as a basis for the making of decisions in a company, providing a strategic guide for leveraging and reflecting process quality (JEANG *et* CHUNG, 2009). The indices are dimensionless measures used to quantify the relationship between process performance and specification limits, and this quantification is essential for the success of improvement activities (WU *et al*., 2009). As a general rule, the larger the value of the index, the better the process is meeting specifications (COSTA *et al.*, 2005).

The four most well-known and widespread basic indices are: C_p, C_{pk}, C_{pm} and C_{pmk}, represented by equations (1), (2), (3) and (4), respectively (DELERYD, 1999; KOTZ and JONHSON, 2002). These indices are used for cases in which the target value (T) is equal to half the length of the specification range (M) and for normally distributed data.

$$\hat{C}_p = \frac{LSE - LIE}{6\hat{\sigma}} \tag{1}$$

$$\hat{C}_{pk} = min\left(\hat{C}_{pI};\ \hat{C}_{pS}\right) \tag{2}$$

$$\hat{C}_{pm} = \frac{LSE - LIE}{6\sqrt{\hat{\sigma}^2 + (\bar{y} - T)^2}} \tag{3}$$

$$\hat{C}_{pmk} = min\left(\hat{C}_{pmI};\ \hat{C}_{pmS}\right) \tag{4}$$

In equations (1), (2), (3) and (4) the upper specification limit, the lower specification limit and the target value of a process are represented respectively by LSE, LIE and T. The mean is represented by $\bar{y}$ and $\hat{\sigma}$ is the estimator of standard deviation.

The C_p index (index of potential capability) considers the process variability without considering the location of the mean, so it is often not applied, since it does not reflect the impact that mean changes have on capability (PEARN *et* KOTZ, 2006). As the C_p index can only be used in processes with bilateral specification limits, Kane (1986) presents the C_{pI} and C_{pS} indices for processes with unilateral specification limits, which are used for evaluation of capability of processes with response variables of greater-is-better and better-is-better types, represented, respectively, in equations (5) and (6). Aiming at assessing the impact that mean changes have on capability, Kane (1986) introduced the C_{pk} index (effective capability index), which assesses both the variability and the centralization of process. Application of C_p and C_{pk} indices in parallel provides good indication of process capability in relation to mean and variability; however, they do not consider the target value of process (T).

$$\hat{C}_{pI} = \frac{\bar{y} - LIE}{3\hat{\sigma}} \tag{5}$$

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$$\hat{C}_{pS} = \frac{LSE - \bar{y}}{3\hat{\sigma}} \tag{6}$$

Based on the quadratic loss function of Taguchi, Chan *et al.*,(1988) and Pearn *et al.* (1992) introduced the C_{pm} and C_{pmk} indices, respectively, which consider both the standard deviation of process and the square of difference between the process mean and the target value. Similarly to C_{pI} and C_{pS} indices, Chan *et al.*,((1988) present the C_{pmI} and C_{pmS} unilateral capability indexes, which must be used for processes with response variables of smaller-is-better and greater-is-better types, respectively, represented by equations (7) and (8).

$$\hat{C}_{pmI} = \frac{\bar{y} - LIE}{3\sqrt{\hat{\sigma}^2 + (\bar{y} - T)^2}} \tag{7}$$

$$\hat{C}_{pmS} = \frac{LSE - \bar{y}}{3\sqrt{\hat{\sigma}^2 + (\bar{y} - T)^2}} \tag{8}$$

When the process means moves away from the target value, the value of C_{pmk} is more sensitive than the values of C_p, C_{pk} and C_{pm} indices (CHANG, 2009). According to Pearn ~~et al.~~ *et al.*,(1992), $C_{pmk} \leq C_{pm} \leq C_{pk} \leq C_p$ and in the equality among these four indicators the process is centralized. If these indices are equal and greater than 1, the process is able of meeting the specifications (VÄNNMAN, 1995).

The indices shown in equations (1), (2), (3), (4) are used for cases in which the target value (T) is equal to the midpoint of the specification limits (M). However, situations in which T≠M occur frequently in productive processes (WU *et al.*,2009). These situations denote asymmetric specification limits, in which the deviations from the target are less tolerable in one direction than in another.

Kane (1986) adapted the C_p, C_{pI}, C_{pS} and C_{pk} indices for asymmetric limits, represented by C^*_p, C^*_{pI}, C^*_{pS} and C^*_{pk}, presented in equations (9), (10), (11) and (12) respectively.

$$\hat{C}^*_p = \frac{min(LSE - T; T - LIE)}{3\hat{\sigma}} \tag{9}$$

$$\hat{C}^*_{pI} = \frac{T - LIE}{3\hat{\sigma}}\left(1 - \frac{|T - \bar{y}|}{T - LIE}\right) \tag{10}$$

$$\hat{C}^*_{pS} = \frac{LSE - T}{3\hat{\sigma}}\left(1 - \frac{|T - \bar{y}|}{LSE - T}\right) \tag{11}$$

$$\hat{C}^*_{pk} = min\ (\hat{C}^*_{pI}, \hat{C}^*_{pS}) \tag{12}$$

Chan *et al.*,(1988) present the C^*_{pm} index, which is a generalization of the C_{pm} index for processes with non-symmetrical specification limits. This index is presented in equation (13). According to Chan *et al.*,(1988), this index relates the smallest difference between the specification limits and the process target values with variability and mean deviations in relation to the process target.

$$\hat{C}^*_{pm} = \frac{min(LSE - T; T - LIE)}{3\sqrt{\hat{\sigma}^2 + (\bar{y} - T)^2}} \tag{13}$$

Pearn *et al.*,(1999) proposed a generalization of C_{pmk} index for processes with asymmetric specification limits. This generalization is defined by equation (14).

$$\hat{C}''_{pmk} = \frac{d^* - A^*}{3\sqrt{\hat{\sigma}^2 + A^2}} \tag{14}$$

where:

$A = max(d(\bar{y}-T) / D_S; d(T-\bar{y}) / D_I)$

$A^* = max(d^*(\bar{y}-T) / D_S; d^*(T-\bar{y}) / D_I)$

$D_S = LSE-T$

$D_I = T-LIE$

$d^* = min(D_S; D_I)$

$d = (LSE-LIE)/2$

Among the studies of capability indices for asymmetric specification limits, the works of Chen *et al.* (1999), Jessenberger *et* Weihs (2000), Pearn *et al.*,((2005), Pearn *et al.*,((2006), Chang *et* Wu (2008) *et* Chang (2009) are the highlights.

The application of capability indices for processes with symmetric specification limits in processes with asymmetric limits might create erroneous conclusions. This same situation occurs in the application of indices presented in non-normal processes, making evident the importance of understanding the behavior and characteristics of each

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productive process. Thus, in non-normal processes, it is necessary to seek alternatives that use capability indices for appropriate distributions (GONÇALEZ *et* WERNER, 2009).

2.2. Regression control charts

Regression control charts are used when the response variable varies due to frequent changes of control variables, not allowing the application of a traditional control chart, which assumes that data is independent and identically distributed around a constant mean. A regression chart should be used in processes in which the effect of a dependent variable is a linear function of an independent variable, given by a linear equation (JACOBI *et al.*, 2002; SHU *et al.*,2004), since the variables involved in the process are correlated and individual control of these variables is not the best choice. Woodall and Montgomery (1999) state that regression charts are one of the techniques developed in the literature with great potential for practical application. One of the objectives of such charts is to control the mean of the response variable due to the control variable setting, unlike traditional control charts that monitor mean.

For this, it is necessary to analyze the relationship between the measured characteristics aiming at finding a quantitative expression that shows these relations. The appropriate model must allow interpreting the situation, getting estimates and making predictions. Application of the modeling technique by linear regression to a group of data results in the determination of linear coefficients, weighting the effect of independent variables on the dependent variable (FOGLIATTO, 2000). The linear regression model is shown in equation (15), representing the relationship between the dependent variable *y* and the *k* independent variables *X* (MONTGOMERY *et al.*, 2001).

$$\boldsymbol{y} = \boldsymbol{X\beta} + \boldsymbol{\varepsilon} \tag{15}$$

where:

$$\boldsymbol{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \qquad \boldsymbol{X} = \begin{bmatrix} 1 & x_{11} & \cdots & x_{1k} \\ 1 & x_{21} & \cdots & x_{2k} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & x_{n1} & \cdots & x_{nk} \end{bmatrix}$$

$$\boldsymbol{\beta} = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_k \end{bmatrix} \qquad \boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix}$$

In equation (15), ***y*** is the vector of values of the response variable, ***X*** is the matrix of values of n observations of *k* control variables, **β** is the vector of regression coefficients, and β_0 is called intercept coefficient (value of *y* when all control variables are equal to zero.) Vector **ε** is estimated by the vector of residuals ***e***, defined as the difference between the values observed and the values estimated by model ($\hat{\boldsymbol{y}}$), presumed to be independently distributed, with zero mean and constant variance σ^2. These assumptions are important for validating the estimated model (NETER *et al.*, 2005). When the number of observations (*n*) is greater than the number of controlled variables (*k*), the method used to estimate the regression equation is the ordinary least squares method, which aims at minimizing the quadratic sums of regression residuals (NETER *et al.*, 2005).

According to Woodal *et* Montgomery (1999), a method of applying control charts can be divided into two phases: *(i)* Phase I (retrospective analysis), which includes the estimation of parameters and *(ii)* Phase II, which includes the monitoring of the process. Once the model is estimated, the regression control chart must be built for Phase I. To build the chart, the upper control limit, the central line and the lower control limit are given by equations (16), (17) and (18) respectively (PEDRINI *et* CATEN, 2011).

$$LSC_i = \hat{y}_i + L\sqrt{QMR} \tag{16}$$

$$LC_i = \hat{y}_i \tag{17}$$

$$LIC_i = \hat{y}_i - L\sqrt{QMR} \tag{18}$$

Where L is a constant value determined according to the sensitivity and number of false alarms wanted for this chart; in most cases, 2 or 3 is adopted. Mandel (1969) adopted 2 standard deviations as a criterion. The estimate of variance of residuals of the regression model is given by the mean square of residuals (QMR), shown in equation (19).

$$QMR = \frac{e'e}{n-p} = \frac{SQR}{n-p} \tag{19}$$

where $\boldsymbol{e} = \boldsymbol{y} - \hat{\boldsymbol{y}}$ e $p = k + 1$

With the process under control in Phase I, the regression chart of Phase II is then developed. It is necessary to presume that the data of the process to be monitored have the same behavior of the data used in Phase I. Data collection for monitoring should contain the response variable monitored, and the respective values of the control variables of the process should be collected at regular time intervals.

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Pedrini *et* Caten's proposal (2011) presents a modification of the control chart based on regression models proposed by Haworth (1996), allowing direct monitoring of the observations referring to a response variable dependent on one or more control variables, instead of monitoring the regression residuals. To build the control chart of Phase II is necessary to correct the control limits of Phase I, since the samples are different. According to Pedrini *et* Caten's proposal (2011), the upper control limit, the center line and the lower control limits are calculated from equations (20), (21) and (22) respectively.

$$LSC_i = \hat{y}_i + 3\sqrt{QMR(1+h_{ii})} \quad (20)$$

$$LC_i = \hat{y}_i \quad (21)$$

$$LIC_i = \hat{y}_i - 3\sqrt{QMR(1+h_{ii})} \quad (22)$$

where $h_{ii}=\mathbf{x}'_i(\mathbf{X}'\mathbf{X})^{-1}\mathbf{x}_i$ and x_i is the vector of the control variables of the *i*-th new observation.

The h_{ii} element is used as a correction factor of the standard deviation of the prediction of a new observation, since it measures the distance of the control variables vector in relation to the vector composed of the mean value of each control variable.

If a point is not under control (above or below the upper and lower limits), the process is considered out of statistical control; in this case, special causes should be investigated and actions should be taken for improvement.

An evolution of the main studies about regression control charts is shown in Figure 1. Loredo *et al.*,(2002) applied the individual measurement chart for residuals of a multiple linear regression model and Shu *et al.*,(2004) proposed the EWMAREG chart, which basically consists in monitoring the standardized residuals of the regression model with an EWMA control chart. Control charts proposed by Haworth (1996), Loredo *et al.* (2002) and Shu *et al.*, (2004) present the further advantage of preserving the temporal order of data, which makes the application of these procedures and the interpretation of results easier, if compared to the method presented by Mandel (1969). A review on regression control charts can be found in Shu *et al.*, (2007).

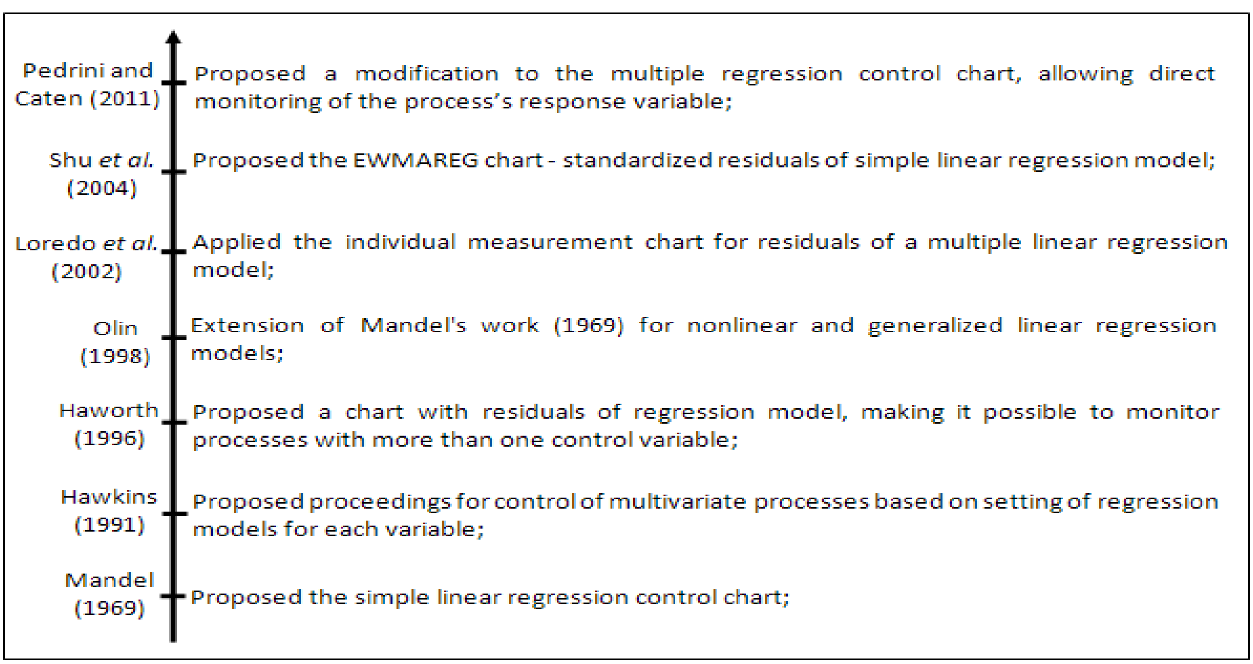

Figure 1 – Evolution of studies concerning control charts based on regression models.

3. METHODOLOGICAL PROCEDURES

This paper involved three steps: *(i)* proposal of capability indices for control charts based on regression models (RC); *(ii)* development of regression chart for a process with random data, and *(iii)* comparison of indices proposed with traditional indices.

The first stage involved the proposal of the RC capability indices, since the application of traditional capability indices in processes monitored with regression control charts may generate erroneous conclusions regarding the capability

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of the process to produce items not complying with specifications.

The proposition of RC capability indices started from the assumption that the target value of the process and the upper and lower control limits in the regression control charts are not constant values as in traditional control charts, but values defined according to the setting of control variables. The indices proposed considered processes with symmetric specification limits, represented by C_{pR}, C_{pkR}, C_{pmR} and C_{pmkR}, and processes with asymmetric specification limits, represented by C^*_{pR}, C^*_{pkR}, C^*_{pmR} and C^*_{pmkR}.

The second phase focused on the development of regression control chart for a process regression with random data based on the method proposed by Pedrini and Caten (2011), which includes the Phase I of data collection, setting of regression model and calculating the control limits of chart of Phase I, and Phase II for process monitoring aiming at identifying and eliminating the special causes resulting from failures in operation. New process data are collected in Phase II, and the monitoring control chart is developed according to the model validated in Phase I.

The third step focused on the application of RC indices and traditional indices by applying those to the simulated process. As soon as the process is stable, the calculation of the RC capability indices is carried out, with the use of standard deviation calculated by equation (23).

$$\hat{\sigma}_R = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{n}} \tag{23}$$

Because it is a calculation of residuals of a new dataset, deducting the estimated value for a regression model whose parameters were calculated with data from Phase I, it is not necessary to deduct the *p* degrees of freedom in the denominator, thus justifying equation (23).

The work method's steps, meeting the development of the regression control chart based on Pedrini and Caten's method (2011), and the calculation of RC indices are summarized in Figure 2.

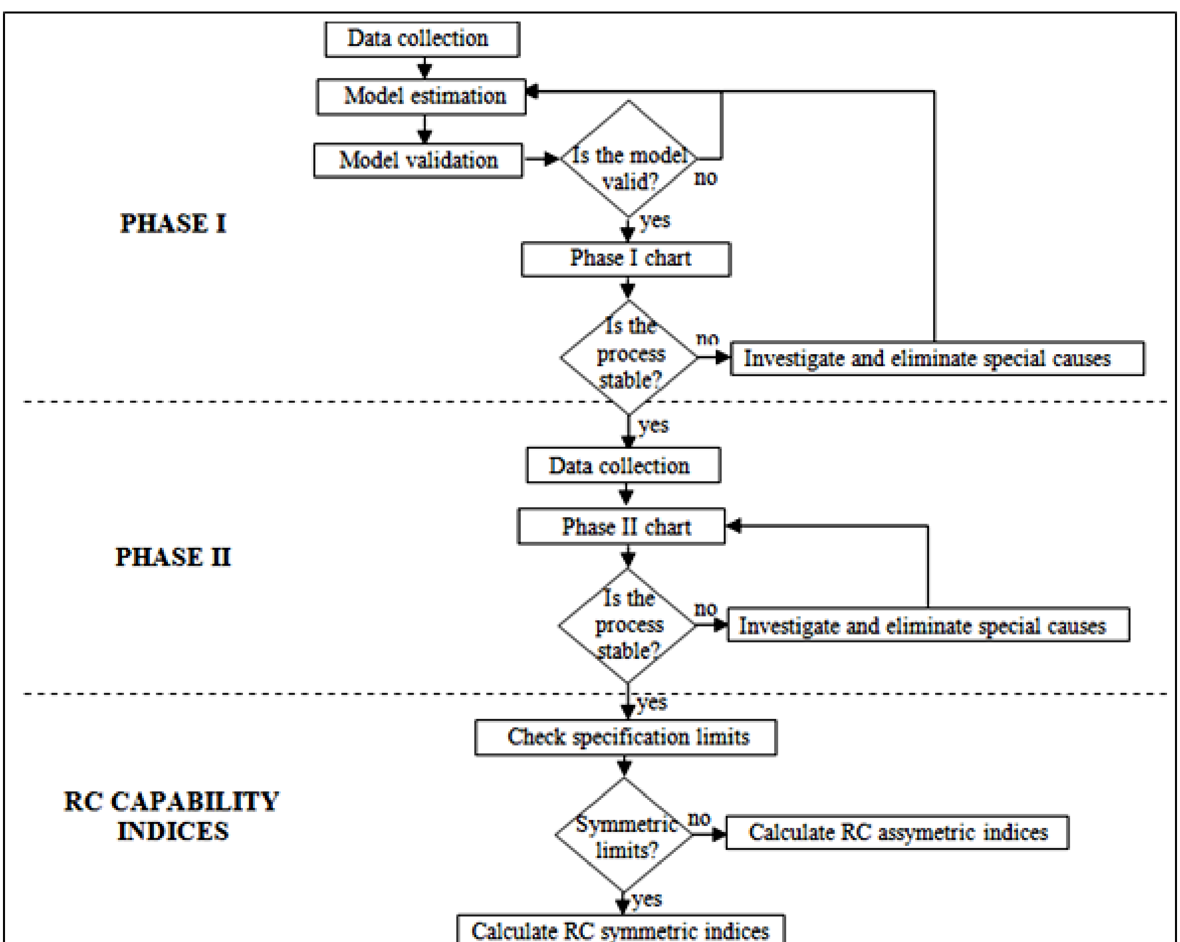

Figure 2 – Method for building regression charts and calculating capability indices.

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4. PROPOSAL OF CAPABILITY INDICES FOR CONTROL CHARTS BASED ON REGRESSION MODELS

In order to develop the RC capability indices, it was necessary to define some concepts. The target value of the process (T) in regression control charts is not a constant value as in traditional control charts, but a value set according to the setting of control variables, as in equation (24). The same is true for the upper specification limit (LSE) and the lower specification limit (LIE), which are calculated by equations (25) and (26). The upper and lower specification limits, as well as the target value, are usually proposed by management or customers from needs and product and/or process analyses.

$$T_i = b_{0T} + b_{1T}x_{1i} + b_{2T}x_{2i} + \cdots + b_{kT}x_{ki} \quad (24)$$

$$LSE_i = b_{0S} + b_{1S}x_{1i} + b_{2S}x_{2i} + \cdots + b_{kS}x_{ki} \quad (25)$$

$$LIE_i = b_{0I} + b_{1I}x_{1i} + b_{2I}x_{2i} + \cdots + b_{kI}x_{ki} \quad (26)$$

where: *i = 1, 2,..., n*

Constants of target intercept, upper limit and lower limit are represented by b_{0T}, b_{0S} e b_{0I}, respectively. In most cases, it is assumed that T, LSE and LIE are parallel, so, $b_{jT} = b_{jI} = b_{jS} = b_k$. The only difference corresponds to the intercept constants, where $b_{oI} < b_{oT} < b_{oS}$, as shown in Figure 3.

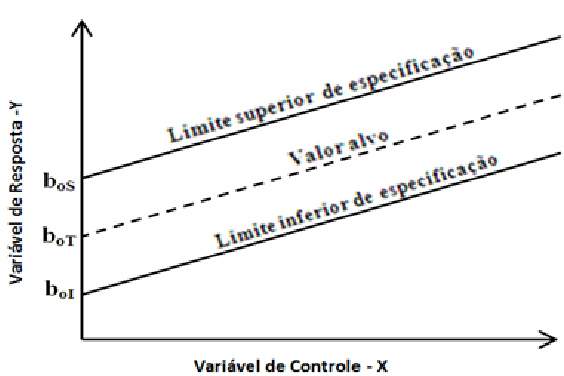

Figure 3 – Identification of specification limits and intercept constants.

4.1. RC CAPABILITY INDICES FOR PROCESSES WITH SYMMETRIC SPECIFICATION LIMITS

Considering that specification limits vary according to the setting of control variables and assuming that LSE_i and LIE_i can be represented by equations (25) and (26), the numerator of the equation of the C_p traditional index is rewritten as in equation (27).

$$LSE - LIE = \frac{\sum_{i=1}^{n} LSE_i - LIE_i}{n} \quad (27)$$

The number of samples is given by n. For simplification, it is assumed that these specification limits are parallel, so $LSE_i - LIE_i$ is equal to $b_{0S} - b_{0I}$. Thus, according to this equation, the difference between $LSE_i - LIE_i$ is always a constant value. By using this result and the constant sum property, equation (28) is obtained.

$$\frac{\sum_{i=1}^{n} LSE_i - LIE_i}{n} = \frac{n(b_{0s} - b_{0I})}{n} = b_{0S} - b_{0I} \quad (28)$$

The C_{pR} potential capability index for regression control charts is shown in equation (29), obtained by substituting equation (28) in equation (1) and using the $\hat{\sigma}_R$ estimated standard deviation shown in equation (23).

$$\hat{C}_{pR} = \frac{b_{0s} - b_{0I}}{6\hat{\sigma}_R} \quad (29)$$

Similarly, the adaptation of the C_{pk} effective capability index for control charts based on regression models is developed by modifying the numerator of the lower capacity index (C_{pI}) and the numerator of the upper capability index (C_{pS}), as in equations (30) and (31).

$$\bar{y} - LIE = \frac{\sum_{i=1}^{n}(y_i - LIE_i)}{n} \quad (30)$$

$$LSE - \bar{y} = \frac{\sum_{i=1}^{n}(LSE_i - y_i)}{n} \quad (31)$$

The substitution of equations (30) and (31) in equations of traditional C_{pI} and C_{pS} indices shown in equations (5) and (6) result in the C_{pkR} effective capability index for control charts based on regression models. The C_{pIR}, C_{pSR} and C_{pkR} indices are shown in equations (32), (33) and (34) respectively.

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$$\hat{C}_{pIR} = \frac{\sum_{i=1}^{n}(y_i - LIE_i)}{3n\hat{\sigma}_R} \quad (32)$$

$$\hat{C}_{pSR} = \frac{\sum_{i=1}^{n}(LSE_i - y_i)}{3n\hat{\sigma}_R} \quad (33)$$

$$\hat{C}_{pkR} = min\left(\hat{C}_{pIR}\,,\hat{C}_{pSR}\right) \quad (34)$$

As previously mentioned, the traditional C_{pm}, C_{pmI} and C_{pmS} indices take into consideration the target value of the process. The denominators of these indices, represented by equations (3), (7) and (8) respectively, are obtained from the Taguchi quadratic loss function, where the target value of the process is constant, as shown in equation (35). However, for processes monitored by control charts based on regression models, the target varies depending on the control variables. Thus, the variance is calculated according to equation (36).

$$\sigma'^2 = E(y_i - T_i)^2 = \hat{\sigma}^2 + (\bar{y} - T)^2 \quad (35)$$

$$\sigma'^2 = E(y_i - T_i)^2 = \frac{\sum_{i=1}^{n}(y_i - T_i)^2}{n} \quad (36)$$

By substituting the equations (28) and (36) in equation (3) of the C_{pm} capability index, the C_{pmR} capability index is obtained. Similarly, the substitution of equations (30), (31) and (36) in equations (7) and (8) creates the C_{pmIR} and C_{pmSR} indices. The capability indices applied to control charts based on C_{pmR}, C_{pmIR}, C_{pmSR} and C_{pmkR} regression models are shown in equations (37), (38), (39) and (40).

$$\hat{C}_{pmR} = \frac{b_{0s} - b_{0I}}{6\sqrt{\frac{\sum_{i=1}^{n}(y_i - T_i)^2}{n}}} \quad (37)$$

$$\hat{C}_{pmIR} = \frac{\sum_{i=1}^{n}(y_i - LIE_i)}{3n\sqrt{\frac{\sum_{i=1}^{n}(y_i - T_i)^2}{n}}} \quad (38)$$

$$\hat{C}_{pmSR} = \frac{\sum_{i=1}^{n}(LSE_i - y_i)}{3n\sqrt{\frac{\sum_{i=1}^{n}(y_i - T_i)^2}{n}}} \quad (39)$$

$$\hat{C}_{pmkR} = min\left(\hat{C}_{pmIR}\,,\hat{C}_{pmSR}\right) \quad (40)$$

4.2. RC CAPABILITY INDICES FOR PROCESSES WITH ASYMMETRIC SPECIFICATION LIMITS

The proposal of capability indices for asymmetric limits, i.e., T≠M, follows the same logic of indices proposed for symmetric tolerances.

The adaptation of C^*_p, C^*_{pI} and C^*_{pS} indices for control charts based on regression models is developed by modifying the numerator, as in equations (41), (42) and (43).

$$T - LIE = \frac{\sum_{i=1}^{n}(T_i - LIE_i)}{n} = b_{0T} - b_{0I} \quad (41)$$

$$LSE - T = \frac{\sum_{i=1}^{n}(LSE_i - T_i)}{n} = b_{0S} - b_{0T} \quad (42)$$

$$T - \bar{y} = \frac{\sum_{i=1}^{n}(T_i - y_i)}{n} \quad (43)$$

By substituting equations (41), (42) and (43) in equations (9), (10), (11) and (12), the RC capability indices are obtained, represented by C^*_{pR}, C^*_{pIR}, C^*_{pSR} and C^*_{pkR} for asymmetric specification limits, according to equations (44), (45), (46) and (47) respectively.

$$\hat{C}^*_{pR} = \frac{min(b_{0S} - b_{0T};\ b_{0T} - b_{0I})}{3\,\hat{\sigma}_R} \quad (44)$$

$$\hat{C}^*_{pIR} = \frac{b_{0T} - b_{0I}}{3\hat{\sigma}_R}\left(1 - \frac{\left|\sum_{i=1}^{n}(T_i - y_i)\right|}{n(b_{0T} - b_{0I})}\right) \quad (45)$$

$$\hat{C}^*_{psR} = \frac{b_{0S} - b_{0T}}{3\hat{\sigma}_R}\left(1 - \frac{\left|\sum_{i=1}^{n}(T_i - y_i)\right|}{n(b_{0S} - b_{0T})}\right) \quad (46)$$

$$\hat{C}^*_{pkR} = min\left(\hat{C}^*_{pIR};\hat{C}^*_{pSR}\right) \quad (47)$$

By substituting equations (36), (41) and (42) in equation (13) of C^*_{pm} traditional capability index for asymmetric limits, the C^*_{pmR} capability index is obtained, shown in equation (48).

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$$\hat{C}^*_{pmR} = \frac{min(b_{0S} - b_{0T};\ b_{0T} - b_{0I})}{3\sqrt{\frac{\sum_{i=1}^{n}(y_i - T_i)^2}{n}}} \quad (48)$$

For the C^*_{pmkR} index for asymmetric specification limits, the elements of equation (14) were modified, represented by equations (49) and (50). The C^*_{pmkR} index is given by equation (51).

$$A_R = max\left[d_R\left(\frac{\sum_{i=1}^{n}(y_i - T_i)}{nD_{SR}}\right);\ d_R\left(\frac{\sum_{i=1}^{n}(T_i - y_i)}{nD_{IR}}\right)\right] \quad (49)$$

$$A_R^* = max\left[d_R^*\left(\frac{\sum_{i=1}^{n}(y_i - T_i)}{nD_{SR}}\right);\ d_R^*\left(\frac{\sum_{i=1}^{n}(T_i - y_i)}{nD_{IR}}\right)\right] \quad (50)$$

$$\hat{C}^*_{pmkR} = \frac{d_R^* - A_R^*}{3\sqrt{\hat{\sigma}_R^2 + A_R^2}} \quad (51)$$

Where:

$D_{SR} = b_{0S} - b_{0T}$

$D_{IR} = b_{0T} - b_{0I}$

$d^*_R = min(D_{SR};\ D_{IR})$

$d_R = (b_{0S} - b_{0I})/2$

5. COMPARATIVE ANALYSIS BETWEEN TRADITIONAL CAPABILITY INDICES AND RC CAPABILITIES INDICES

The process under study is composed of a response variable and four control variables (x_1, x_2, x_3 and x_4) and monitored by control charts based on regression models. In Phase I, the estimated regression model presented in equation (52) was based on a project of experiments, showing a QMR = 70.06 and a coefficient of determination R^2 of 85.2%. In this model, the interactions were not included, since they had p-values greater than the significance level adopted (0.05).

$$\hat{y}_i = 89.84 + 9.49x_1 - 5.13x_2 - 6.48x_3 + 12.59x_4 - 7.94x_1^2 \quad (52)$$

With the assumptions of validity in the model presented in Pedrini *et* Caten's method (2011), the control chart based on regression models is developed for Phase II of process monitoring meeting 100 new samples, presented in the Appendix. Figure 4 shows the control chart of Phase II.

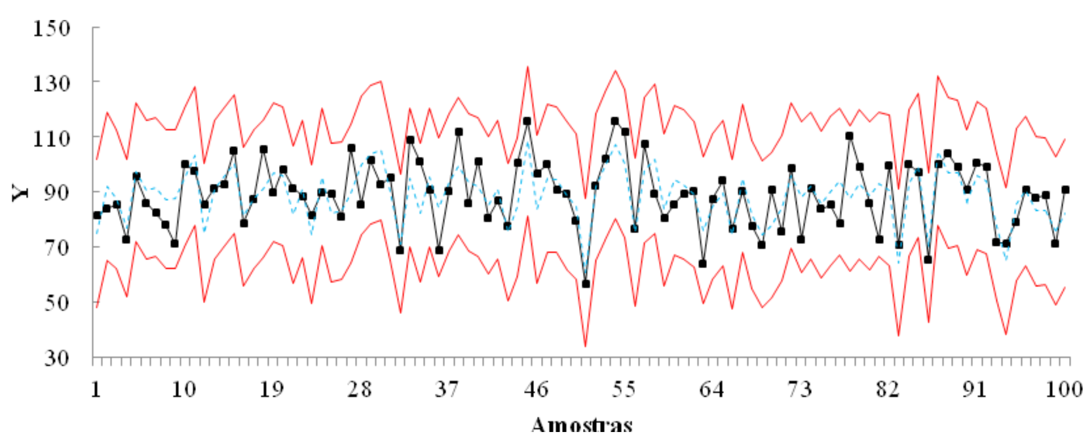

Figure 4 – Control chart based on regression models for process monitoring.

The control chart shown in Figure 4 is under statistical control, since it did not indicate samples outside the region defined by the control limits. Thus, it is possible to carry out the analysis of the process' capability. The standard deviation for calculating the capability indexes is obtained using equation (23).

To calculate the capacity indices, the following were considered: *(i)* symmetric specification limits, and *(ii)* asymmetric specification limits.

To calculate the symmetric traditional capability indices, LSE, LIE and T were considered as worth respectively 160, 50 and 105. For the calculations of the symmetric RC indices, the specification limits and the target value vary according to the control variables, as in equations (53), (54) and (55).

$$LSE_i = 160.00 + 9.49x_1 - 5.13x_2 - 6.48x_3 + 12.59x_4 - 7.94x_1^2 \quad (53)$$

$$LIE_i = 50.00 + 9.49x_1 - 5.13x_2 - 6.48x_3 + 12.59x_4 - 7.94x_1^2 \quad (54)$$

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$$T_i = 105.00 + 9.49x_1 - 5.13x_2 - 6.48x_3 + 12.59x_4 - 7.94x_1^2 \quad (55)$$

For the application of asymmetric traditional indices, T = 80 was set as the process' target value, having the same values considered for LSE and LIE. To calculate the RC asymmetric indices, T was defined according to equation (56), with the same equations (53) and (54) being considered for LSE and LIE.

$$T_i = 80.00 + 9.49x_1 - 5.13x_2 - 6.48x_3 + 12.59x_4 - 7.94x_1^2 \quad (56)$$

To calculate the symmetric traditional indices (T=M) shown in Table 1, equations (1), (2), (3), (4), (5), (6), (7) and (8) were used, and for the symmetric RC indices, equations (29), (32), (33), (34), (37), (38), (39) and (40) were used. For the asymmetric traditional indices (T≠M) shown in Table 2, equations (9), (10), (11), (12), (13) and (14) were used, and for the asymmetric RC indices, equations (44), (45), (46), (47), (48) and (51) were used. Calculations of indices are presented in the Appendix. Figure 5 shows the comparison of indices for symmetric and asymmetric limits.

Table 1 – Traditional capability indices and RC indices for control charts based on regression models considering symmetric specification limits (T=M).

Symmetric specification limits							
	Traditional indices	$\hat{C}_p$	$\hat{C}_{pI}$	$\hat{C}_{pS}$	$\hat{C}_{pk}$	$\hat{C}_{pm}$	$\hat{C}_{pmk}$
		1.47	1.04	1.89	1.04	0.91	0.64
	RC indices	$\hat{C}_{pR}$	$\hat{C}_{pIR}$	$\hat{C}_{pSR}$	$\hat{C}_{pkR}$	$\hat{C}_{pmR}$	$\hat{C}_{pmkR}$
		2.22	1.50	2.95	1.50	0.83	0.56

Table 2 – Traditional capability indices and RC indices for control charts based on regression models considering asymmetric specification limits (T≠M).

Asymmetric specification limits							
	Traditional indices	$\hat{C}_p^*$	$\hat{C}_{pI}^*$	$\hat{C}_{pS}^*$	$\hat{C}_{pk}^*$	$\hat{C}_{pm}^*$	$\hat{C}_{pmk}''$
		0.80	0.56	1.89	0.56	0.65	0.64
	RC indices	$\hat{C}_{pR}^*$	$\hat{C}_{pIR}^*$	$\hat{C}_{pSR}^*$	$\hat{C}_{pkR}^*$	$\hat{C}_{pmR}^*$	$\hat{C}_{pmkR}^*$
		1.21	1.07	3.09	1.07	0.71	0.83

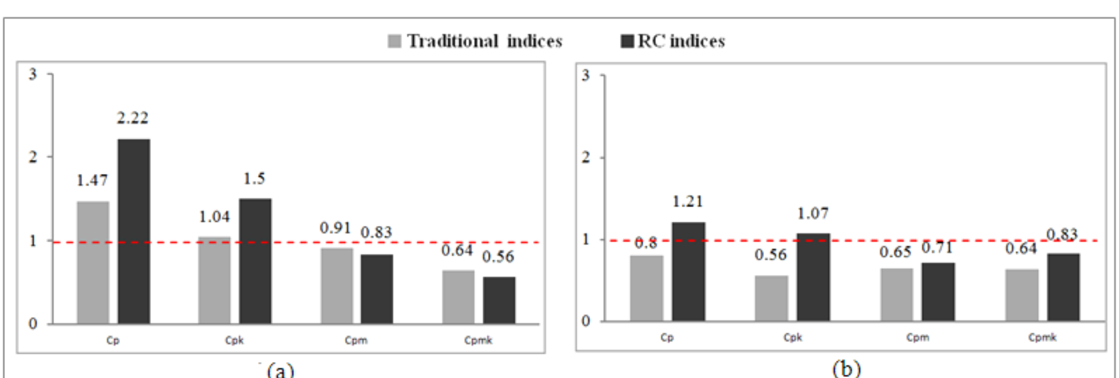

Figure 5 – (a) Comparison of traditional and RC capability indices for symmetric limits; and (b) Comparison of traditional and RC capability indices for asymmetric limits.

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It is possible to see in Table 1 and Figure 5a that the C_p index had a value of 1.47, inferior to the value found for the C_{pR} index, which was 2.22, meaning a process with excellent capability to meet specifications. The C_{pkR} index also shown a superior value in relation to the C_{pk} traditional index. It is seen that the values obtained for C_{pI} and C_{pIR} indicate that the process has a mean (89.5) closer to the lower specification limit, since the values obtained were inferior if compared to those of the C_{pS} and C_{pSR} indices. The C_{pkR} index had a value of 1.50, meaning that the process is capable. However, it will be able to increase its capability by the means of centralization, since the C_{pR} potential capability index indicates a capability of 2.22. The C_{pm} and C_{pmk} indices, as well as the C_{pmR} and C_{pmkR} indices, refer to the process as non-capable, since it does not reach the T target.

According to Table 2 and Figure 5b, the C^*_p capability index had a value of 0.80, indicating that the process is potentially not capable. However, the C^*_{pR} index showed a value of 1.21, meaning the process is potentially capable. The same situation occurs with C^*_{pk} (0.56) and C^*_{pkR} (1.07) indices. The C^*_{pmR} and C^*_{pmkR} indices present low values, showing the process as not able to reach the T target, having similar behavior to the same indices for symmetric limits shown in Table 1.

Therefore, it is seen that traditional indices used in processes monitored with control charts based on regression models may lead to erroneous conclusions. In this context, it is relevant to stress how important it is to correctly apply the capability indices according to the type of control chart used to monitor the process.

6. FINAL THOUGHTS

This study aimed at: *(i)* proposing RC capability indices for control charts based on regression models, considering symmetric and asymmetric specifications; and *(ii)* comparing RC capability indices with traditional capability indices by applying those in a simulated process.

The proposed RC capability indices assume that specification limits are not fixed, but vary depending on the setting of control variables. Based on this assumption, adjustments were made in the traditional capability indices and the C_{pR}, C_{pkR}, C_{pmR} and C_{pmkR} indices were proposed for symmetric specifications, and the C^*_{pR}, C^*_{pkR}, C^*_{pmR} and C^*_{pmkR} indices were proposed for asymmetric specifications.

Comparisons were made between the values of traditional indices and those of the RC indices proposed, by applying them in a process with random data. Misused indices can create erroneous conclusions, risking the study and analysis of process, and thus jeopardize the meeting of requirements from management or external customers.

As a suggestion for future studies, the application of RC indices into productive processes is recommended, including estimation steps of the control chart based on regression models. The development of a flowchart is also recommended, aiming at guiding the choices of capability indices according to the characteristics of each process.

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APPENDIX

Table 3 presents the database in which the capability indices were applied.

Table 3 – Database

	x_1	x_2	x_3	x_4	y	$\hat{y}$		x_1	x_2	x_3	x_4	y	$\hat{y}$
1	37.30	4.29	16.46	122.64	81.39	74.92	*51*	27.91	7.62	15.80	109.15	56.66	60.81
2	58.36	6.42	14.55	231.48	83.95	92.04	*52*	109.60	4.35	14.71	134.58	92.56	91.84
3	83.93	5.21	9.89	104.06	85.63	87.59	*53*	73.75	6.60	1.70	198.65	102.11	99.55
4	51.28	6.76	12.81	127.82	72.54	76.98	*54*	82.02	4.91	7.83	245.68	115.80	107.27
5	81.21	5.73	4.55	167.19	95.61	97.29	*55*	68.52	4.65	4.75	189.35	111.64	100.31
6	120.32	4.47	14.83	135.21	86.00	90.85	*56*	31.09	5.73	11.77	154.80	76.70	75.52
7	103.10	5.39	8.76	124.50	82.51	91.81	*57*	104.79	4.04	15.90	182.64	107.37	97.99
8	76.23	4.98	7.79	95.70	78.13	87.30	*58*	96.26	7.99	2.17	225.85	89.45	101.96
9	136.10	7.59	15.25	192.94	71.19	87.52	*59*	114.47	4.59	19.79	101.93	80.36	83.60
10	85.96	5.58	13.09	185.74	100.27	95.10	*60*	57.55	6.82	0.55	187.39	85.64	94.29
11	88.36	5.28	12.94	240.95	97.60	103.20	*61*	55.00	5.89	6.60	193.68	89.47	92.75
12	38.24	6.57	12.48	146.83	85.61	75.11	*62*	109.61	5.11	16.85	138.27	90.25	88.98
13	70.06	2.77	19.07	147.55	91.56	90.91	*63*	36.86	6.28	10.52	144.12	64.04	76.18
14	78.14	5.75	6.77	168.00	92.66	95.40	*64*	78.99	6.64	9.66	115.28	87.62	84.76
15	102.75	4.47	4.13	148.70	105.27	100.23	*65*	97.57	7.33	11.25	161.55	94.19	89.75
16	116.90	6.26	10.57	71.02	78.65	81.24	*66*	107.62	6.58	17.24	55.95	76.56	74.64
17	73.86	4.56	18.84	149.14	87.45	87.49	*67*	41.50	5.78	6.21	245.21	90.53	94.92
18	70.86	4.75	15.44	169.02	105.62	91.08	*68*	110.28	6.82	19.29	128.00	77.72	81.69
19	97.74	5.63	14.68	203.47	89.65	97.16	*69*	35.73	4.61	9.34	96.15	70.64	74.69
20	102.00	5.90	-0.34	120.64	98.34	95.90	*70*	37.13	5.21	5.13	107.42	90.85	77.91
21	110.01	4.87	14.90	66.90	91.51	81.86	*71*	72.59	6.28	12.20	127.30	75.60	84.36
22	121.22	3.79	4.41	70.63	88.65	91.12	*72*	108.04	4.58	9.53	145.12	98.90	95.98
23	50.39	7.00	13.79	120.88	81.65	74.55	*73*	122.42	5.95	3.03	85.39	72.99	88.22
24	103.74	4.95	9.68	149.49	90.08	95.50	*74*	86.49	4.62	1.26	82.06	91.58	92.25
25	41.90	3.50	12.16	128.59	89.42	82.40	*75*	120.21	4.93	12.39	89.59	83.82	85.52
26	53.77	5.00	17.68	160.14	81.11	83.23	*76*	68.56	5.04	3.63	112.05	85.39	90.31
27	104.13	6.06	9.56	124.05	105.99	89.53	*77*	113.65	5.17	19.74	195.72	78.69	94.01
28	72.41	5.16	4.99	190.11	85.65	99.79	*78*	118.52	6.24	4.25	89.19	110.54	87.55
29	66.08	4.36	7.77	229.79	101.55	103.60	*79*	86.24	5.09	12.21	153.43	99.18	92.90
30	87.40	4.20	6.06	199.58	92.65	105.12	*80*	66.35	7.04	17.69	215.56	86.02	88.56
31	81.98	5.06	5.27	99.07	95.26	90.07	*81*	68.01	6.02	16.82	221.09	72.98	92.83
32	40.59	5.76	20.90	135.03	68.74	71.21	*82*	110.51	5.19	13.55	136.57	99.57	90.67
33	64.27	5.83	11.83	216.82	109.09	95.09	*83*	51.51	6.42	19.20	53.60	70.70	64.46
34	118.10	6.07	14.94	99.60	100.95	82.39	*84*	61.48	6.53	3.46	179.41	100.28	93.27
35	98.71	4.81	4.40	119.25	90.78	95.35	*85*	116.63	5.43	12.01	207.37	97.08	99.61
36	83.76	5.25	13.28	97.20	68.99	84.42	*86*	43.03	4.71	12.40	52.24	65.64	69.98
37	78.98	3.10	6.20	94.88	90.58	93.51	*87*	121.70	4.31	2.54	181.92	100.03	104.97
38	100.72	5.45	5.09	167.86	112.02	99.47	*88*	78.04	7.99	8.51	235.31	103.89	96.98
39	118.20	3.52	11.90	120.69	85.76	93.56	*89*	95.85	5.05	10.05	168.61	98.94	97.17
40	81.00	7.68	11.77	201.36	101.15	91.86	*90*	44.41	5.20	14.06	195.29	90.91	86.18
41	40.59	4.90	12.97	187.93	80.61	85.21	*91*	64.34	4.68	13.65	210.43	100.84	96.08
42	75.00	5.18	16.98	178.65	86.80	91.04	*92*	108.51	5.59	5.38	127.43	99.07	93.84
43	67.70	5.68	12.72	55.26	77.54	75.42	*93*	59.06	4.96	13.95	79.29	71.78	77.21
44	74.57	4.90	17.22	124.07	100.55	84.63	*94*	25.21	5.31	13.50	94.71	71.36	65.05
45	114.30	5.97	3.52	243.88	115.69	108.49	*95*	33.05	7.97	6.93	247.90	79.19	85.54
46	32.27	4.37	10.90	183.94	96.95	83.81	*96*	68.40	7.30	0.19	141.54	91.08	90.42
47	108.38	4.28	4.25	105.01	100.14	95.12	*97*	92.19	4.51	17.84	87.97	88.16	83.10
48	67.07	5.10	6.26	162.09	90.65	94.39	*98*	88.18	5.79	10.06	76.72	88.87	83.07
49	61.50	7.65	6.01	179.95	89.63	88.82	*99*	77.14	6.59	10.84	54.69	71.21	76.17
50	51.22	5.33	4.95	120.35	79.54	84.80	*100*	30.29	4.25	13.79	192.21	90.88	82.36

Tables 4 and 5 present the equations of RC capability indices applied to data for symmetric and asymmetric specification limits respectively.

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Table 4 – RC capability indices for symmetric specification limits.

RC capability indices for symmetric limits	*Application of RC indices in the simulated process*	*Results*
$\hat{C}_{pR} = \frac{b_{0S} - b_{0I}}{6\hat{\sigma}_R}$	$\hat{C}_{pR} = \frac{160 - 50}{6x8.24}$	**2.22**
$\hat{C}_{pIR} = \frac{\sum_{i=1}^{n}(y_i - LIE_i)}{3n\hat{\sigma}_R}$	$\hat{C}_{pIR} = \frac{3699.09}{3x100x8.24}$	**1.50**
$\hat{C}_{pSR} = \frac{\sum_{i=1}^{n}(LSE_i - y_i)}{3n\hat{\sigma}_R}$	$\hat{C}_{pSR} = \frac{7300.91}{3.100x8.24}$	**2.95**
$\hat{C}_{pkR} = min(\hat{C}_{pIR}, \hat{C}_{pSR})$	$\hat{C}_{pkR} = min(1.50\ ;\ 2.95)$	**1.50**
$\hat{C}_{pmR} = \frac{b_{0S} - b_{0I}}{6\sqrt{\frac{\sum_{i=1}^{n}(y_i - T_i)^2}{n}}}$	$\hat{C}_{pmR} = \frac{160 - 50}{6\sqrt{\frac{49112.51}{100}}}$	**0.83**
$\hat{C}_{pmIR} = \frac{\sum_{i=1}^{n}(y_i - LIE_i)}{3n\sqrt{\frac{\sum_{i=1}^{n}(y_i - T_i)^2}{n}}}$	$\hat{C}_{pmIR} = \frac{3699.09}{3x100\sqrt{\frac{49112.51}{100}}}$	**0.56**
$\hat{C}_{pmSR} = \frac{\sum_{i=1}^{n}(LSE_i - y_i)}{3n\sqrt{\frac{\sum_{i=1}^{n}(y_i - T_i)^2}{n}}}$	$\hat{C}_{pmSR} = \frac{7300.91}{3x100\sqrt{\frac{49112.51}{100}}}$	**1.10**
$\hat{C}_{pmkR} = min(\hat{C}_{pmIR}, \hat{C}_{pmSR})$	$\hat{C}_{pmkR} = min(0.86, 1.85)$	**0.56**

Table 5 – RC capability indices for asymmetric specification limits.

RC capability indices for asymmetric limits	*Application of RC indices in the simulated process*	*Results*
$\hat{C}^*_{pR} = \frac{min(b_{0S} - b_{0T};\ b_{0T} - b_{0I})}{3\hat{\sigma}_R}$	$\hat{C}^*_{pR} = \frac{min(160 - 80;\ 80 - 50)}{3x8.24}$	**1.21**
$\hat{C}^*_{pIR} = \frac{b_{0T} - b_{0I}}{3\hat{\sigma}_R}\left(1 - \frac{\lvert\sum(T_i - y_i)\rvert}{n(b_{0T} - b_{0I})}\right)$	$\hat{C}^*_{pIR} = \frac{80 - 50}{3x8.24}\left(1 - \frac{360.96}{100(80 - 50)}\right)$	**1.07**
$\hat{C}^*_{pSR} = \frac{b_{0S} - b_{0T}}{3\hat{\sigma}_R}\left(1 - \frac{\lvert\sum(T_i - y_i)\rvert}{n(b_{0S} - b_{0T})}\right)$	$\hat{C}^*_{pSR} = \frac{160 - 80}{3x8.24}\left(1 - \frac{360.96}{100(160 - 80)}\right)$	**3.09**
$\hat{C}^*_{pkR} = min(\hat{C}^*_{pIR}; \hat{C}^*_{pSR})$	$\hat{C}^*_{pkR} = min(1.07; 3.09)$	**1.07**
$\hat{C}^*_{pmR} = \frac{min(b_{0S} - b_{0T};\ b_{0T} - b_{0I})}{3\sqrt{\frac{\sum(y_i - T_i)^2}{n}}}$	$\hat{C}^*_{pmR} = \frac{min(160 - 80;\ 80 - 50)}{3\sqrt{\frac{19768.00}{100}}}$	**0.71**
$\hat{C}^*_{pmkR} = \frac{d^*_R - A^*_R}{3\sqrt{\hat{\sigma}^2_R + A^2_R}}$	$\hat{C}^*_{pmkR} = \frac{30 - 3.61}{3\sqrt{67.96 + 6.62^2}}$	**0.83**