



## Optimization Model for Cross-Functional Decision Making: A Computational Business Intelligence Approach

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### ABSTRACT

Latest manufacturing technologies enhance cross-functional interaction between manufacturing and marketing. In spite of increasingly emphasizing on the aspect of end user's demand, many production decision-making processes do not take into account only the dynamic nature of the marketer. Here, an attempt has been made to bridge the gap between marketing and partially integrated production problem, with the objective of developing mathematical model that can act as an optimizer in an add-on advanced planning system within an enterprise. Basic idea of this research is the integration of work on determining production and raw material batch sizes under different ordering and delivery assumptions for heuristically evaluating the two-stage batch production problem. Production rate is considered to be a decision variable. Integrated unit production cost function is formulated by considering the various pertinent factors. Proposed model is developed simultaneously by formulating constrained maximization problem for marketing division and minimization problem for production division. Considering the complexities for highly non-linear optimization problem, a Computational Intelligence approach is successfully developed and implemented. The model is practical in nature and may be used as an add-on optimizer that co-ordinates distinct function with an aim of maximizing the profit function in any firm.

**Keywords:** *Two - stage batch production system; Variable demand; Cross functional decision making; Optimization; Elitist real coded genetic algorithm*

## INTRODUCTION

Manufacturing and marketing are the main functions of any business firm where harmony should be sustained in order to keep successful business operations (Park & Hong, 2009). However, managing manufacturing / marketing interface is extremely crucial since the two functions assist and reinforce each other (Chen & Chu, 2003). Therefore, these two functions have adverse and most of the time conflicting priorities that have to be dealt by management for integrative decision making.

In any conventional production system, all decisions regarding cross functional decision making are taken at a time by the decision maker. But, for a large manufacturing firm with multidimensional objective, this policy will not be implemented: they are to assist production and marketing decisions by considering all the pertinent factors. Marketing and Production department are interlinked, so the conflicts are inevitable. Marketing is considered as 'creation of consumer demand' whereas Operations is considered as 'fulfillment of consumer demand'. Here, marketing division first analyzes their strategy and take decisions regarding their products, forecast demand and volume of production. According to the target as fix-up by the marketing division, production department try to implement that target in the best possible optimized way. Problem of this type in cross functional decision - making is popularly known as Partially Integrated Production and Marketing (PIPM) Problem.

Many inventory models in concerned to PIPM have been proposed to deal with a variety of real life complex inventory problems (Pal et al. 2007). Inventory is an important resource in supply chains, serving many functions and taking many forms. But, like any resource it must be managed well if an organization is to remain competitive. High inventory levels 'hide' problems - while lowering inventory exposes problems. The challenge in inventory control, is not only to pare inventory to the bone to reduce costs or to have plenty around to satisfy all demand, but to have the right amount available to achieve the competitive priorities for the business most efficiently. Inventory models considering the effect of demand and incorporating the marketing decisions are highlighted by many researchers (Abad, 1994; Bhunia & Maiti, 1994; Bhunia et al. 2009; Goyal & Gunasekaran, 1995; Urban, 1992). An inventory model of deteriorating items with lot-size dependent replenishment cost and a linear trend in demand is highlighted by Bhunia & Maiti (1999). Research areas in PIPM model, one may refer to the work of Lee and Kim (1993). They developed a PIPM model for integrated production and marketing planning



under the consideration that items are sold after getting appropriate lot.

Most of the researchers have used traditional optimization techniques for solving production - inventory / supply chain problems. The traditional methods of optimization and search do not fare well over a broad spectrum of problem domains. Traditional techniques are not efficient when the practical search space is too large. These algorithms are not robust. Numerous constraints and number of passes make the production / supply chain problem more and more complicated. Traditional optimization techniques such as geometric programming, dynamic programming, branch and bound techniques and quadratic programming found it hard to solve these problems and they are inclined to obtain a 'local' optimal solution. To overcome these limitations, there has been a growing interest in optimization algorithms and methods: popularly termed as Computational Intelligence. Recently, this type of methods, such as Genetic Algorithms, Simulated Annealing, Particle Swarm, Tabu Search etc. have been used by the researchers for parameter optimization, classification and learning in a wide range of applications of managerial decision making problems. According to Goldberg (1989), Davis (1991), Michalewicz (1992), and Sakawa (2002) Genetic algorithms are adaptive computational procedures which are modeled as the mechanics of natural genetic systems. They express their abilities by efficiently exploiting the historical information to speculate on new offspring with expected improved performance. The GA searches populations of solutions of an optimization problem towards improvement by simulating the natural search and selection process associated with natural genetics. Genetic Algorithm is different from traditional optimizations in the following ways:

- Works with a coding of the parameter set and not the parameters themselves.
- Searches from a population of points and not limited to a single point.
- Uses information of fitness function and not derivatives or other auxiliary knowledge.
- Uses probabilistic transition rules rather than deterministic rules.

In each iteration, three basic genetic operations i.e., selection, crossover and mutation are performed. Prof. J. H. Holland of University of

Michigan envisaged the fundamental concept of this algorithm in the mid sixties and published his seminal work Holland (1975). In the recent year, a number of researchers have contributed their work and try to implement this methodology in several implicational fields like Traveling salesman problems by Forrest (1993), Scheduling problem by Davis (1991), Numerical optimization by Michalewicz (1992), and many more. But, till now only a very few researchers have applied Computational Intelligence methodologies in the field of production-inventory control model. Extensive research work in this field refers to Sarker and Khan (1999), Sarker and Newton (2002), Sarker & Yao (2003), Roy et al. (2005), Roy et al. (2008), Sarker & Parija (1994), etc.

## **MATHEMATICAL MODEL FORMULATION**

In this paper, we consider a manufacturing firm's production system that procures raw materials from suppliers in a lot and after a series of batch production operation it transformed into a finished product. It is a common practice to evaluate separately the Economic Lot Sizes for manufacturing a product and purchasing raw materials. However, when the raw materials are used in production and their ordering quantities are dependent on the batch quantity of the product : it comes under the limelight of not to isolate the problem of economic purchase of raw materials from economic batch quantity (Wu et al. 2003). Taking jointly the production and purchasing as components of a single integrated system, we determine the optimum production lot size of a product and the ordering quantities of associated raw materials together by minimizing the total cost of the system.

### **Assumptions & Notations**

We develop a mathematical model of the problem. First, a general model is developed by considering both supplier of raw material and buyer of finished products. Using this model we develop an optimal ordering policy for procurement of raw materials and the manufacturing batch size to minimize the total cost for meeting equal shipments of the finished products at fixed intervals to the buyers.

For this development, we consider a manufacturing production system that procures raw materials from suppliers in a lot and after a series of batch production system it turns into finished product. It is a common practice to evaluate separately the Economic Lot Sizes for manufacturing a product and purchasing raw materials. However, when the raw materials are used in



production and their ordering quantities are dependent on the batch quantity of the product it comes under the importance of not to isolate the problem of economic purchase of raw materials from economic batch quantity. Considering production and purchasing as components of a single system we determine the optimum production lot size of a product and the ordering quantities of associated raw materials together by minimizing the total cost of the system.

In raw material inventory situation of any organization, normally the raw materials are consumed only at a given rate during the production up time of the batch and not throughout the total period of cycle. Keeping in mind for the benefit of the firm, management takes the following principles:

1. Ordering policy of the raw materials can either be based on its
  - Economic Order Quantity (EOQ) or
  - On the requirement of the raw material in a lot of Economic Production Quantity
2. Optimum purchasing - production policy should be determined by considering the system as a whole and not by treating its components individually.

The finished product supply policy may be continuous or periodic. Based on the supply pattern of raw material, it can be classified as follows:

- Ordering quantity of raw material is equal to the raw material required for one lot of a product.
- Ordering quantity of raw material is equal to the raw material required for multiple lots of the product.
- Multiple order of a raw material for one lot of product to be manufactured.

The goal of the management is to determine the optimum batch size of the product and the ordering quantities of raw materials that minimizes the overall cost of the system. In this area, research works of Sarker, Karim & Haque (1995), Sarker and Parija (1994), etc. are worth-mentioning. Sarker and Khan (1999) developed an optimal batch size for a production system operating under periodic delivery policy. In their model formulation of option - II (Case - A & Case - B), the integer variables are relaxed as continuous variable and developed an expression using different floating-point parameters. Taking the lower and upper integral values of the integer variables, they solved their

problems. But, this method does not guarantee for finding the optimality of the problem. The said problem should be solved by mixed integer programming methodology for non - linear programming.

In this paper, we develop an elitist real - coded Genetic Algorithm (ERCGA) for mixed integer non - linear programming to investigate the effects of partially integrated production and marketing policy of a profit making firm and also to determine jointly the optimal policy for procurement of raw materials and the manufacturing batch size. Here, the demand rate is assumed to be a function of the marketing cost and selling price of the product. The selling price is determined by a mark - up over the production cost which is a generalized cost function dependent on the several factors like raw material cost, labor charges, production rate and other factors of the manufacturing system. The production rate is finite which is considered to be a decision variable. The model has been formulated for different feasible cases under various scenarios based on the supply pattern of the raw materials and the delivery policy of the product. In each feasible case with different scenarios, constrained maximization problem for marketing division and constrained minimization problem for production division have been developed separately. Mathematical modeling are solved by developing ERCGA with ranking selection, uniform crossover for integer variable, and arithmetical crossover for non-integer variables, mutation and elitism under the platform of Genetic Algorithm. Finally, feasible cases have been illustrated with the help of numerical example.

### Notations

The following notations are used in developing our proposed model.

$P_p$  : Production rate which is taken as decision variable

$D_p$  : Demand rate (unit / year)

$Q_p$  : Production lot size

$H_p$  : Annual inventory holding cost (Rs / unit / year)

$A_p$  : Setup cost for the product (Rs / set-up)

$r_i$  : Quantity of a raw material required in producing one unit of the product (particular raw material  $i; i = 1$ )

$D_1$  : Demand of raw material (particular) for the product in a year,  $D_1 = r_i \cdot D_p$

$Q_1$  : Ordering quantity of raw material (particular,  $i; i = 1$ )

$A_1$  : Ordering cost of a raw material (particular,  $i; i = 1$ )



- $H_1$  : Annual inventory holding cost for raw material (particular,  $i; i = 1$ )  
 $P_{r_i}$  : Price of raw material (particular,  $i; i = 1$ )  
 $Q_1^*$  : Optimum ordering quantity of raw material (particular,  $i; i = 1$ )  
 $x$  : Shipment quantity to customer at a regular interval (units / shipment)  
 $L$  : Time between successive shipments, =  $(x/D_p)$   
 $T$  : Cycle time measured in year =  $(Q_p/D_p)$   
 $m$  : Number of full shipments during the cycle time =  $T/L$   
 $t_s$  : Production time in a cycle

### Assumptions

The following assumptions are used to develop our proposed model.

1. Only one raw material will be used in the proposed production system.
2. The unit production cost  $f(P_p)$  is dependent on raw material cost, labors charges and production rate of the product and is expressed as

$$f(P_p) = P_{r_i} + \frac{L}{P_p^\beta} + k_1 P_p^\gamma$$

$P_{r_i}, L, k_1$  are non-negative real number. Here,  $P_{r_i}$  be the raw material costs (for particular item  $i, i=1$ ) and  $L$  be the labor cost to produce one unit of Product.

3. Demand rate  $D_p(P_p \succ D_p)$  is a function of selling price  $S_p$  and marketing cost  $M$  per unit production. It takes the form:

$$D_p(M, S_p) = M^a (a - b S_p) \text{ where } a, b \text{ being constant}$$

$$S_p = \lambda_p f(P_p); \lambda_p \text{ is the mark - up rate.}$$

4. Shortages are not allowed.
5. Time horizon is infinite.
6. Product cannot be delivered until the whole lot is finished.
7. Supply of raw materials will be in lots and the lot size is a decision variable.
8. Supply policy of finished product is assumed to be continuous / periodic.



9. Number of lot-sizes is assumed to be integers for all the cases.

Based on the supply pattern of the raw material our proposed model can be classified in the following possible scenarios:

Scenario - I: One lot of each raw material will be required for one lot of production.

Scenario- II: One lot of each raw material will be required for multiple lots of production

Scenario - III: Multiple lots of each raw material will be required for one lot of production

Delivery policy of the product can be classified as follows:

(a) Periodic Supply

Whole lot in single installment (Case - A)

Whole lot in multiple installments (Case - B)

(b) Continuous Supply (Case-C)

In Case - A, the whole production lot will be delivered in one shipment or distributed to one customer or among different wholesalers / customers at a time. In Case - B, the whole production lot will be delivered in multiple shipments to one customer or distributed among different wholesalers / customers. However, in Case - C, the product will be delivered or sold continuously from the beginning of the production.

Here, we discuss Scenario - I and Scenario - II separately. But it is very important to note that, **Case - B of Scenario - I and Case - C of Scenario - II are not feasible.**

Marketing division determines the selling price of the final product to the customer based on several expenditure and demand or production volume of that product. Therefore, our prime objective is to maximize the marketing profit  $Z_p$  per unit time. The profit will be determined in the following manner :

**Profit = [Revenue] – [Marketing Cost] – [Production Cost]**

Hence the problem of marketing department is to maximize the marketing profit.

$$\text{Max } Z_p(M, P_p) = S_p D_p - M \cdot D_p - f(P_p) D_p \quad (1)$$

subject to the condition that  $P_p > D_p$



We denote this problem as *marketing sub - problem*. Here, the profit function is a function of two continuous variables  $M$  the marketing expenditure per unit product and  $P_p$ , the production rate per unit time. This problem is common to all the feasible cases of different scenarios. Now, we shall find out the average cost of all the feasible cases for different scenarios.

### Scenario - I: Case - A

In this case, the ordering quantity of raw material is equal to the requirement of the raw material for a batch of the production system. The raw material that is replenished at the beginning of a production cycle will be fully consumed at the end of this production run. Here, the delivery policy of the product is continuous. This system is shown in Figure - 1 and Figure - 2.

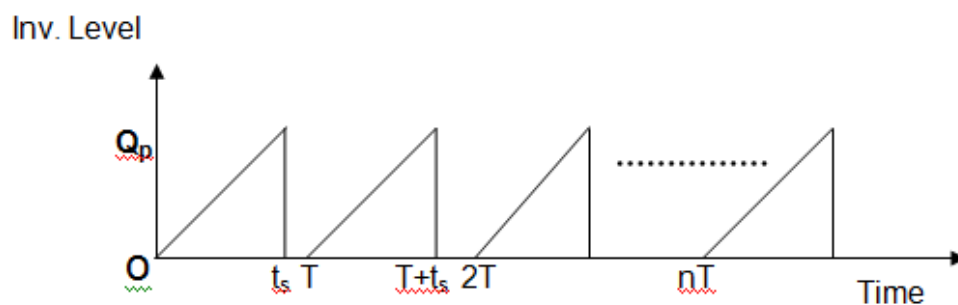


Figure -1: Finished Product Inventory Situation

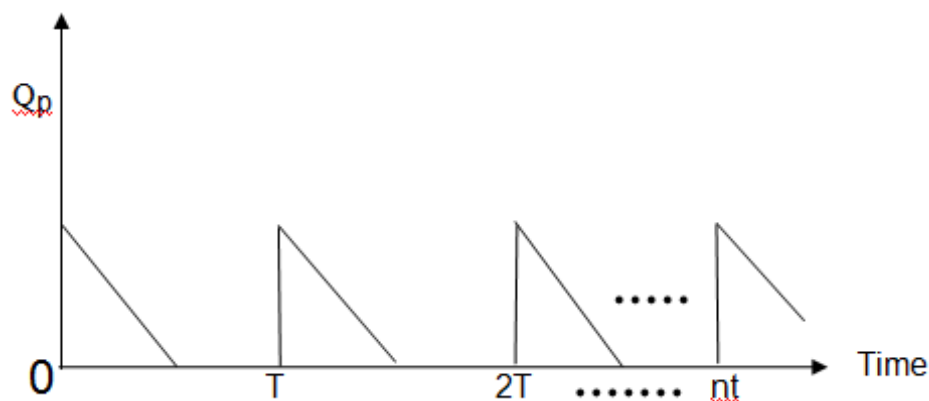


Figure - 2: Raw material inventory situation

The problem of production department is to minimize the production cost. In this case, the average cost of the system ( $i = 1$ ) is equal to the sum of set up cost of the finished product, inventory holding cost of the finished product, ordering cost and inventory costs for raw material.

$$Z_c = \frac{(A_p D_p)}{Q_p} + \frac{H_p(Q_p D_p)}{2P_p} + \frac{(A_1 D_1)}{Q_1} + \frac{H_1(Q_1 D_p)}{2P_p} \quad (2)$$

Based on the demand determined from the problem (1) production division enquires about how much amount to be produced and how much time the production will be continued so that the  $Z_c$  obtained from (2) is minimized.

Hence the sub-problem of production division is to minimize the production cost is as follows

$$Z_c = \frac{(A_p D_p)}{Q_p} + \frac{H_p(Q_p D_p)}{2P_p} + \frac{(A_1 D_1)}{Q_1} + \frac{H_1(Q_1 D_p)}{2P_p} \quad (3)$$

subject to  $Q_p > 0$   $t_s > 0$

Therefore, **NET PROFIT** of the system is given by

$$Z = Z_p - Z_c$$

Problem (1) and (3) can be solved by using any traditional optimization method or any soft computing method in Computational Intelligence platform. However, these optimization methods can get trapped at a 'local' optimum that can be overcome by genetic algorithm.

### Scenario - I: Case - C

In this case, the pictorial representations of raw material and finished product situation are shown in Figure -3 and Figure -4 respectively.

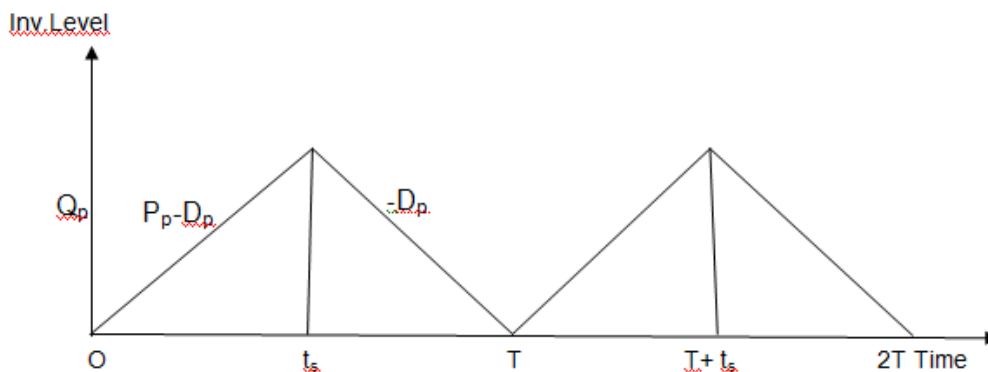


Figure - 3: Finished Product - inventory situation

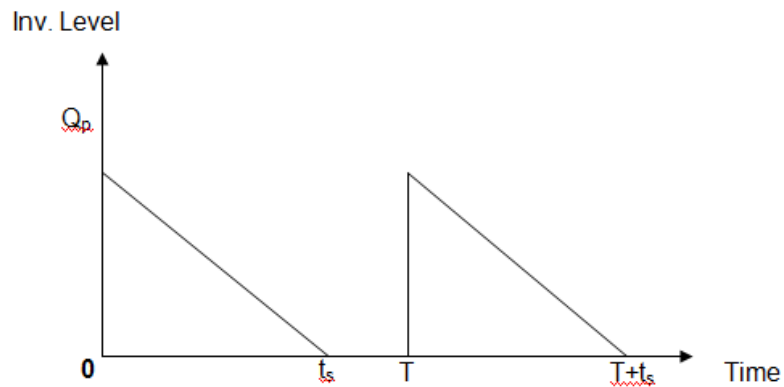


Figure - 4: Raw material inventory situation

In this case, the production sub problem is given by

$$Z_c = \frac{A_p D_p}{Q_p} + \frac{H_p t_s (P - D)}{2} + \frac{A_1 D_1}{Q_1} + \frac{H_1 Q_1 D_p}{2P_p} \quad (4)$$

subject to  $Q_p > 0$  and  $t_s > 0$

Therefore the **NET PROFIT** of the system is given by

$$Z = Z_p - Z_c$$

Again, problem (1) and (4) can be solved by using any traditional optimization or any soft computing method under Computational Intelligence platform. However, these optimization methods can get trapped at a 'local' optimum that can be overcome by genetic algorithm.

### Scenario -II: Case - A

In this case, the pictorial representations of raw material and finished product situation are shown in Figure - 5 and Figure - 6 respectively.

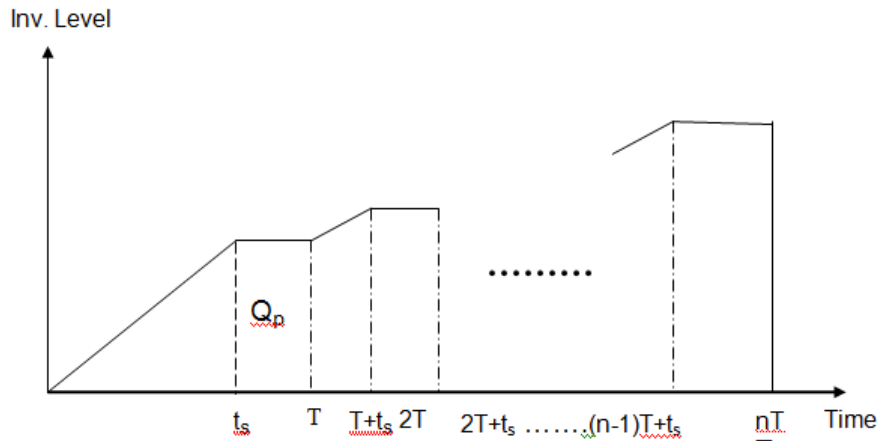


Figure - 5: Production - inventory situation

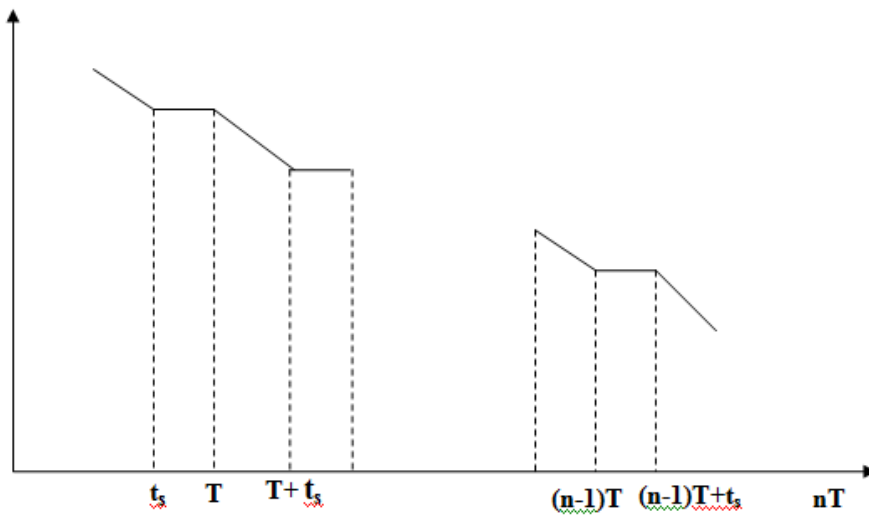


Figure - 6: Raw material situations

In this case, the production sub problem is given by

*Minimize*

$$Z_c = \frac{A_p D_p}{Q_p} + \frac{(n+1) Q_p t_s}{2Q_p} - \frac{A_1 D_1}{2T} + \frac{A_1 D_1}{Q} + \frac{r_1 Q_p H_1}{2} \left( \frac{D_p}{P_p} + n - 1 \right) \tag{5}$$

subject to  $Q_p > 0$  and  $t_s > 0$

Therefore the **NET PROFIT** of the system is given by

$$Z = Z_p - Z_c$$

Again, problem (1) and (5) can be solved by using any traditional optimization or any soft computing method under Computational Intelligence platform. However, these optimization methods can get trapped at a 'local' optimum that can be overcome by genetic algorithm.

### Scenario - II: Case - B

In this case, the pictorial representations of raw material and finished product situation are shown in Figure - 7 and Figure - 8 respectively.

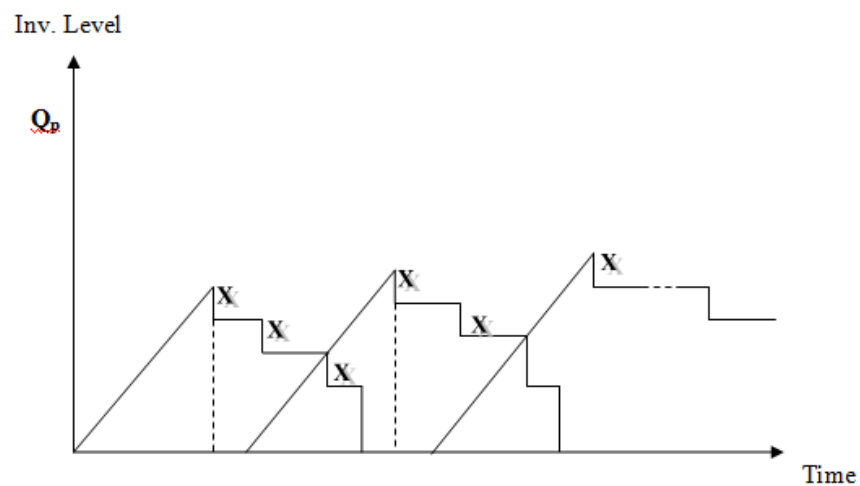


Figure - 7: Production - inventory situations

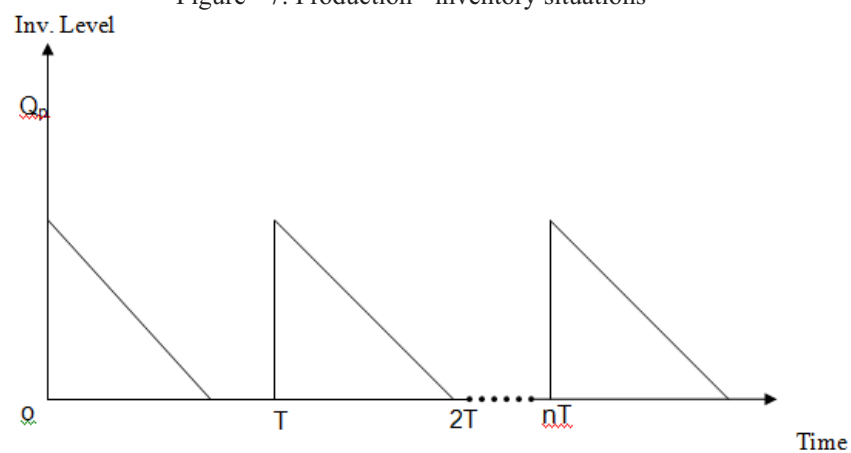


Figure - 8: Raw material inventory system

In this case, the production sub problem is given by

$$Z_c = \frac{D_p}{Q_p}(A_p + A_1) + \left[ \frac{1}{2} Q_p \left( \frac{D_p}{P_p} + 1 \right) - \frac{x}{2} \right] H_p + \frac{Q_p t_s}{2T} \quad (6)$$

$$x = \frac{Q_p}{m}, \quad m = \text{number of full shipments during the cycle time} = \frac{T}{L}$$

subject to  $Q_p > 0$   $t_s > 0$

Therefore, **NET PROFIT** of the system is given by

$$Z = Z_p - Z_c$$

Again, problem (1) and (6) can be solved by using any traditional optimization or any soft computing method under Computational Intelligence platform. However, these optimization methods can get trapped at a 'local' optimum that can be overcome by genetic algorithm.

Now, we shall develop the algorithms for determining the optimal value of  $M$  and  $P_p$  with the average profit for marketing division of the proposed production inventory system by a real coded Genetic Algorithm for solving constrained maximization problem involving two continuous variables and also constrained minimization involving either one continuous variable or one continuous and one discrete variables.

## COMPUTATIONAL INTELLIGENCE METHODOLOGY

### Genetic Algorithm

A Genetic algorithm is a parallel and evolutionary search algorithm based on Darwinian Theory (Holland 1975). It is used to search large, non-linear solution space where expert knowledge is lacking or difficult to encode. Moreover, it requires no gradient information; evolves from one population to another and produces multiple optima rather than single local one. In most cases, GA is conducting its search so that it may optimize some known fitness function (Sakawa, 2002). The way in which the function itself is 'known' is of limited theoretical, but much practical importance. At the extreme, the fitness of a proposed solution could be tested in the physical world. Once, an initial population has been formed, 'selection', 'crossover' and 'mutation' operations are repeatedly performed until the fitness number of evolving population converges to an optimal fitness value. Alternately, the GA may run for a user



defined number of iterations to evolve a strategy of building a good solution instead of finding the solution directly.

The stepwise procedure of Genetic Algorithms is shown as follows:

Step-1: Initialize the parameters of Genetic Algorithm, bounds of variables and different parameters of the proposed inventory system.

Step-2:  $t = 0$  [ $t$  represents the number of current generation]

Step-3: Initialize  $P(t)$  [ $P(t)$  represents the population at  $t$  - th generation]

Step-4: Evaluate  $P(t)$

Step-5: Store 'better' result from  $P(t)$

Step-6:  $t = t + 1$

Step-7: If ( $t >$  maximum generation number) go to step -14

Step-8: Select  $P(t)$  from  $P(t-1)$  by standard selection process

Step-9: Alter  $P(t)$  by crossover and mutation operation

Step-10: Evaluate  $P(t)$

Step-11: Find 'better' result from  $P(t)$

Step-12: Compare optimal results of  $P(t)$  and  $P(t-1)$  and store better one

Step-13: Go to step - 6

Step-14: Print 'better' result

Step-15: Stop

To use a genetic algorithm, six fundamental issues should be determined: chromosome representation, reproduction function, the genetic operators making up the reproduction function, initializing population, termination criteria, and fitness function

### Parameters of Genetic Algorithm

Genetic Algorithm depends on different parameters like population



size (POPSIZE), probability of crossover (PCROS), probability of mutation (PMUTE) and maximum number of generation (MAXGEN). Population size in Genetic Algorithm is problem dependent. If the population is too large, there arise some difficulties in storing of the data for large population size. Generally, the probabilities of crossover and mutation are taken as 0.85 to 0.95 and 0.05 to 0.15 respectively. In our present inventory analysis, we have taken the values of these parameters as follows:

$$POPSIZE = 200, PCROS = 0.92, PMUTE = 0.12, MAXGEN = 200.$$

### Coding

Main problem in applying genetic algorithm is to design an appropriate chromosome representation of solutions of the problem with genetic operators. Coding in GA is the form in which chromosomes and genes are expressed. There are mainly two types of coding : binary and real. The binary coding was presented in the GA in which the chromosome is expressed as a binary string. Therefore, the search space of the considered problem is mapped into a space of binary strings through a coder mapping. Then, after reproducing an offspring, a decoder mapping is applied to bring them back to their real form in order to compute their fitness function values. However, the real coding is more applicable and easy in programming. Moreover, it seems that the real coding fits the continuous optimization problems better than the binary coding. In this paper, a chromosome is coded in the form of a vector / matrix of real numbers, every component of chromosomes represents a variable of the function.

### Initialization

To initialize a population in genetic algorithm, generally POPSIZE number of chromosomes  $V_1, V_2, \dots, V_{popsize}$  are generated randomly. However, it is very much difficult for complex optimization problems to produce feasible chromosome explicitly. Generally, for each chromosome  $V_i$ , every gene is randomly generated within the desired domain randomly in such a way that it should be feasible in nature.

### Evaluation Function

Evaluation function plays exactly the same role in GA as the environment plays in natural evolution. Generally, evaluation function,



$EVAL(X)$  for the chromosome  $X$  is taken equivalent for the objective function  $f(X)$ . After getting a population, our objective is to search a chromosome that gives better value of the objective function. For this, we have to calculate the fitness for each chromosome. The value of the objective function due to the chromosome  $V_i$  is taken as the fitness value of  $V_i$  and it is denoted by  $eval(V_i)$ .

### Selection

Every kind of selection must prefer individuals within a good fitness to such one with a worse. Weak individuals ought to get a though little chance to pass their alleles to the next generation yet. This is very important for the spread out of the individuals over the parameter space; other wise the evolution will make a premature convergence to local optimum, may be. Thus the purpose of selection is to emphasize the better individuals in the population for recombination in hopes that their offspring will in turn have even higher fitness. Selection has to be balanced with variation from crossover and mutation: too strong selection means that sub-optimal highly fit individuals will take over the population, by reducing the diversity needed for further change and progress; too weak selection will result in too - slow evolution.

Several standard techniques are used for selection viz. deterministic sampling method, roulette wheel selection ( $N, \mu$ ) selection, stochastic tournament selection (Wetzel ranking) or Ranking selection method, etc. In ranking method, selection probabilities are calculated normally and successive pairs of individuals are drawn by using roulette wheel selection. Here, we perform ranking selection over roulette wheel where fitness is taken as the value of the objective function.

The algorithm we use for this probabilistic selection process is:

Step - 1: Compute all the fitness  $f_i \forall i = 1, 2, \dots, n$ .

Step - 2: Sort all  $f_1, f_2, \dots, f_n$  in descending order for maximization problem and identify all the sorted  $f_i \forall i = 1, 2, \dots, n$  by marking numbers  $(1, 2, \dots, n)$ .

Step - 3: Generate a random real number  $r_1$  in  $[0, 1]$

Step - 4: Calculate the probability  $p_i \forall i = 1, 2, \dots, n$  for each chromosome

$V_i$  by using the formula  $p_i = r_1(1-r_1)^{i-1}$

Step - 5: Compute the cumulative probability  $P_i$  for each chromosome

$V_i$  by using the formula  $P_i = \sum_{j=1}^i p_j$

Step - 6: Generate a random real number  $r_2$  in  $[0,1]$

Step -7: Obtain the minimal  $k$  such that  $P_k \succ r_2$

Step - 8: Select the  $k$ -th individual.

Step - 9: Repeat step (6) to (8) until the no. of selected individuals coincide with population size.

### Crossover

The exploration and exploitation of the solution space is made possible by exchanging genetic information of the current chromosomes. Crossover operator operates on two parent solutions (chromosomes) at a time and generates offspring by combining both parent solutions features. For this operation, expected  $N$  (the integral value of  $PCROS*POPSIZE$ ) number of solutions will take part. Hence, in order to perform the crossover operation,  $PCROS*POPSIZE$  number of chromosomes are to be selected. For this purpose, we adopt Random Stochastic Sampling scheme (without replacement). After selection of chromosomes, the crossover operation is applied. In this paper, we use non-linear whole arithmetical crossover.

Different steps of crossover operation are given below:

Step -1: Assign  $N$  as the integral value of  $PCROSS*POPSIZE$

Step - 2: Generate a random real number  $r_3$  in  $[0,1]$

Step - 3: Select the chromosome  $V_k$  and  $V_i$  robustly from the population for crossover operation if  $r_3 \prec PCROS$

Step - 4: Generate a proper fraction  $\lambda$  by the formula

$$\lambda = \frac{p_{\max}}{(p_{\max} + p_{\min})}, \quad \text{where} \quad p_{\max} = \max[p_j \forall j = 1, 2, \dots, POPSIZE] \quad \text{and}$$



$$p_{\min} = \min[p_j \forall j = 1, 2, \dots, \text{POPSIZE}]$$

Step - 5: Produce two offsprings  $V_k'$  &  $V_1'$  by

$$V_k' = \lambda V_k + (1 - \lambda) V_1$$

$$V_1' = \lambda V_1 + (1 - \lambda) V_k$$

Step - 10: Repeat Step - 2 and Step - 5 for  $N/2$  times.

### Mutation

Mutation introduces random variations in the population and is used to prevent the search process from converging to local optima rapidly. Mainly, this operation is responsible for fine tuning of the system and is applied to a single chromosome. After introducing genetic diversity and turning the population gently into a slightly better converge way, mutation operation performed with a low probability by defeating the order building as generated via selection and crossover. Here, we shall use non-uniform mutation whose action is dependent on the age of the population. If the element (gene)  $V_{ik}$  of chromosome  $V_i$  is selected for mutation and domain of  $V_{ik}$  is  $[l_{ik}, u_{ik}]$ , then the reduced value of  $V_{ik}$  is represented by

$$V_{ik}' = V_{ik} + \Delta(t, u_{ik} - V_{ik}); \text{ if a random digit is 0,}$$

$$= V_{ik} - \Delta(t, V_{ik} - l_{ik}); \text{ if a random digit is 1.}$$

Where  $k \in \{1, 2\}$  and the function  $\Delta(t, y)$  returns a value in the range  $[0, y]$  such that the value of  $\Delta(t, y)$  being close to 0 as  $t$  increases. This property causes this operator to search the space uniformly initially (when  $t$  is small) and vary locally at later stages.

Here, we have taken

$$\Delta(t, y) = y[1 - r^{(1-t/T)^b}]$$

$r$  is a random number from  $[0, 1]$ ,  $T = \text{MAXGEN}$ ,  $t$  represents the current generation and  $b$  (called non uniform mutation parameter) is constant.

The algorithm of mutation operation is as follows:

Step - 1:  $i = 1$

Step - 2: generate a random number  $r$  from  $[0, 1]$ .

Step - 3: If  $r < PMUTE$ , then select the chromosome  $V_i$  and go to Step -5.

Step - 4:  $i = i + 1$

Step - 5: select a particular gene  $V_{ik}$  of selected chromosome  $V_i$

Step - 6: Create new gene corresponding to the selected gene  $V_{ik}$  by mutation operation.

Step - 7: repeat the steps (1) - (6) for  $PMUTE * POPSIZE$  times.

### Elitism

Real Coded GA technique is generally stochastic in nature. Sometimes, the best chromosome may be lost when a new population is created by crossover and mutation operation. In this sequel, to increase the performance rapidly, one more highly fitted chromosomes (individuals) are considered in the new population to prevent the loss of best-found solution. This is known as elitism and we have incorporated this to get better result.

### Termination

If number of iteration is less than or equal to MAXGEN, then the process is going on; otherwise it terminates. If a termination condition is satisfied, report the 'best' chromosome found so far and stop; o.w construct the new population by taking the rest chromosome in the current population and proceed accordingly.

### NUMERICAL ILLUSTRATION

To solve this problem, a real-coded GA for integer variables has been developed. To illustrate the developed model, let us consider a hypothetical system with the following values of parameters. As there is no real-world data available due to commercial confidentiality and neither is there any benchmark data available from the literature, the values considered here are feasible in the proposed system. All algorithms were coded in Microsoft Visual C++ version



6.0 and all experiments were run on a PC Pentium IV 1.8 GHZ with 256 RAM running Microsoft Windows 2000 professional Version 5. All algorithms started from a solution produced by a greedy heuristic and allowed fairly computation time for a better comparison.

$$P_{r_1} = 40, L = 1000, a = 1000, b = 0.5, \alpha = 0.1, \beta = 1.5, \gamma = 0.5, k_1 = 0.2, \lambda = 1.25,$$

$$A_p = 500, A_1 = 1000, h_p = 10.5, h_1 = 7.0, r_1 = 1.0, x = 500, M = 1.0376$$

For the above parametric values, the sub problem (constrained maximization problem) of marketing department and the sub problem (constrained minimization problem) of production department are solved by real coded elitist GA methods. In 20 runs, the best -found results are shown in Table -1

#### Result of marketing division (Maximization)

$$P_p = 1488.8701, Z_p = 10610.06, D_p = 973.7565, f(P_p) = 47.7346, S_p = 59.6682$$

#### Result of production division (Minimization)

Option - I / Case - A (Minimization)

$$[ Popsiz = 200, Maxgen = 200, Q_p = (450, 600) ]$$

$$Q_p = 505.05, Z_c = 5782.310$$

Option - I / Case - C (Minimization)

$$[ Popsiz = 200, Maxgen = 200, Q_p = (550, 600) ]$$

$$Q_p = 596.75, Z_c = 4897.5816$$

Option - II / Case - A (Minimization)

$$[ n(lower) = 1, upper = 20 ]$$

$$Q_p = 300.1098, Z_c = 9735.5947$$

Option - II / Case - B (Minimization)

$$[ n(lower) = 1, upper = 20 ]$$

$$Q_p = 798.8794, Z_c = 3674.4231$$

Table - 1: Best found solutions for different options by Genetic Algorithm

| Scenario | I          |            | II         |            |
|----------|------------|------------|------------|------------|
|          | Case - A   | Case - C   | Case - A   | Case - B   |
| $Z_p$    | 10610.0600 | 10610.0600 | 10610.0600 | 10610.0600 |
| $Z_c$    | 5782.3100  | 4897.5816  | 9735.5947  | 3674.4231  |
| $Z$      | 4827.7500  | 5712.4784  | 874.4653   | 6935.6369  |
| $M$      | 1.0376     | 1.0376     | 1.0376     | 1.0376     |
| $P_p$    | 1488.8701  | 1488.8701  | 1488.8701  | 1488.8701  |
| $f(P_p)$ | 47.7346    | 47.7346    | 47.7346    | 47.7346    |
| $S_p$    | 59.6682    | 59.6682    | 59.6682    | 59.6682    |
| $D_p$    | 973.7565   | 973.7565   | 973.7565   | 973.7565   |
| $Q_p$    | 505.0500   | 596.7500   | 300.1098   | 798.8794   |
| $n$      | -          | -          | 1          | 1          |

In applying Genetic Algorithm there is a probable chance of arising difficulty regarding the boundaries of the decision variables. Generally, in application problems, selection of boundaries of the decision variables is a hard task. We have overcome these difficulties by selecting the decision variables randomly and taking the minimum value of the corresponding capacity and requirement of the source and destination respectively. By this process, all constraints are satisfied accordingly.

Here it should be noted that, though the above table presents the relatively 'Best' solutions found for the various feasible cases, but a table of mean or mode performance may give a better idea of the algorithms overall performance. Sensitivity analysis is envisaged for further study.

## CONCLUSION

In today's rapidly changing business environment, manufacturing and





marketing interface be carefully managed in order to run successful business operations. Despite a growing interest in this topic, there has been a lack of research about this issue both in operation management and marketing literature.

In this paper, we have developed an optimization model for two-stage batch production problems by taking into consideration the finite rate of production for determining jointly an optimal ordering policy for procurement of raw materials and the manufacturing batch size. For cross functional decision making for marketing as well as production division, decision has been taken separately by formulating maximization problems for marketing division and minimization problem for production division. For solution of this complex two-stage production system, first time elitist GA with real coding under Computational intelligence platform is developed and successfully implemented. However, for two cases - Option - II (Case - A & Case - B) to solve the minimization problem, we have developed mixed integer programming for two floating point variables and one integer type variable. Regarding the selection boundaries for the decision variables, there arise some difficulties: we try to overcome it by trial and error method. The solution proposed by our approach is compared to the effectiveness of the lower bound of the cost and make span in order to prove the qualities and robustness of our proposed approach. The GA solutions are always better than the solutions found by the company. As many metaheuristics, the GA might find better results in more than one hour of execution time. Needless to say the company's solution made use of the experience of the scheduler, but not the GA's solution. Viewed as a tool to support decision-making, the GA seems to provide a good starting point from which the scheduler can improve lot sizes and schedules for line and tanks. It is worth mentioning that a scheduler normally takes two days to conclude a production plan.

For future work, this model may be solved by using advanced methodologies like Simulated Annealing, Ant Colony Optimization and Particle Swarm Optimization under Computational Intelligence platform. Again, hybridization of two methods like Simulated Annealing with Genetic Algorithms (SAGA) may come out with even more 'better' results. Finally, this optimization model may be implemented in different industries and particularly directed to business intelligence provider with the recent use of mathematical modeling as an aid to computational intelligence platform.

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