Volume 2, Number 1, 2005, pp. 39-56

Using Geostatistics to Estimate the Variability of Autocorrelated Processes

Sueli Aparecida Mingoti
Department of Statistics, Federal University of Minas Gerais (UFMG),
Belo Horizonte, MG, Brazil
E-mail: sueli@est.ufmg.br

Otaviano Francisco Neves
Federal University of Minas Gerais (UFMG),
Belo Horizonte, MG, Brazil

Abstract

Statistical quality control is used to detect changes in the parameters values of the process which usually are estimated under the assumption of independence of the sampling units with respect to the quality characteristic. However, this is questionable for many processes. The main objective of this paper is to present estimators for the variance of autocorrelated processes by using Geostatistics methodology. With this new procedure the usual Shewhart's control charts still can be used to monitor the quality of the process. A Monte Carlo simulation study showed that the proposed estimators have good performance.

Keywords: variability, autocorrelation, geostatistics, semivariogram, shewhart's control charts

Introduction

The quality of a process is usually monitored by control charts. Basically, they are a representation of the quality characteristic measured in a sample or in several samples of the process. These charts define an area where the values of the quality characteristic (X), or its average, should stay for the process to be considered under control. For continuous inspection the chart for average contains a central line (CL) that represents the average value of the quality characteristic and two horizontal lines called lower and upper control limits (LCL; UCL) calculated under the assumption that X has a normal distribution. Sample points outside the limits are an indication that the process is "out of control" (Montgomery, 2001). As a consequence, there is always a probability of rejecting the "under control condition" of the process erroneously, which is defined as "false alarm". This is the case where the sample points, or averages, fall outside the control limits due to

Volume 2, Number 1, 2005, pp. 39-56

the randomness of the normal distribution and not due the fact that some modification of the process parameters had occurred. Under the normality assumption the control limits for the average of the process are given by the following equations:

$$UCL = \mu + k\frac{\sigma}{\sqrt{n}};\ CL = \mu;\ LCL = \mu - k\frac{\sigma}{\sqrt{n}} \tag{1}$$

where μ and σ are the average and the standard deviation of X, respectively and k is the distance of the control limits to the central line expressed in units of standard deviation.

In practice the values of μ and σ are estimated from samples of the process, when it is just under the effect of "common" or "random" causes. Let $X_1, X_2, \ldots, X_n$ be the observed values of a random sample of the process. Then the parameter μ is estimated by the sample mean $\overline{X}$ and the parameter σ is estimated by the standard deviation (s) or the moving sample range ($\hat{\sigma}_{AM}$) defined respectively as

$$s = \left[\frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2\right]^{1/2} \quad \text{where} \quad \overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \tag{2}$$

$$\hat{\sigma}_{AM} = \frac{\overline{AM}}{1.128}, \text{ where } \overline{AM} = \frac{\sum_{i=2}^{n} AM_i}{n-1} \text{ and } AM_i = |X_i - X_{i-1}| \tag{3}$$

The variance of the process (σ^2) is estimated by the square of the estimators (2) and (3), respectively.

These classical estimation procedures are based on the assumption of independence among the sample units of the process with respect to the quality characteristic X being measured. As mentioned in Alwan and Roberts (1995) this assumption is very questionable especially for chemical processes (Zhang, 1998). With the advances of the technology, processes can be sampled at higher rates which often leads to autocorrelated data. When the estimators (2) or (3) are used to estimate the standard deviaton σ of an autocorrelated process then the chance of "false alarms" or not detecting the "out of control" condition may increase because the calculated control limits will be shorter or wider than the true limits of the process. According to Zhang (1998) and Box and Luceno (1997) positive correlation occurs more frequently in practical situations.

A decrease in the correlation effect can be achieved by increasing sampling units interval. However, this alternative can increase the time needed to detect that the process is "out of control" and for some continuous processes of production it can not be implemented. Corrections of control limits when the correlation is intrinsically part of the process are being proposed in the literature by several authors using time series models (Alwan and Roberts, 1989; Runger and Willemain, 1995). One of these alternatives is the identification and adjustment of the ARIMA model (Box and Jenkins, 1976) for the series of the process observations. After the adjustment, the residuals of the model are obtained and Shewhart's control charts are constructed to the series of residuals, which by assumption should be independent and identically distributed according to a normal distribution. The possible changes that could happen in the average of the process would be reflected in the behavior of the residuals (Box and Luceno, 1997). Although very interesting, this alternative demands the right identification of the ARIMA model and the calculation of the residual for each new collected sample. Another alternative is to monitor the process by using the *EWMA* (Exponentially Weighted Moving Average) charts proposed iniatially by Roberts (1959) and discussed by Montgomery and Mastrangelo (1991), Hunter (1998, 1986) among others. Basically the statistical *EWMA* model is defined as

$$Z_t = \lambda X_t + (1 - \lambda) Z_{t-1} \tag{4}$$

where $0 \leq \lambda \leq 1$ is a constant which needs to be determined by the user and X_t is the value of the quality characteristic X observed for the sample t, $t = 1,2,\ldots,n$. By using the model (4) the series of the one step forecasts errors is obtained and Shewhart's control charts are then applied to the series of errors which theoretically should be uncorrelated. The choice of the constant λ in (4) is discussed in Crowder (1989), Lucas and Saccucci (1990), Box and Luceno (1997) among others. Basically, it is chosen to minimize the sum of squares of the one step forecasts prediction errors. Hunter (1998) claimed that the *EWMA* control chart is simpler to implement and can be an efficient tool to be used in companies daily routine. Another approach proposed by Krieger, Champ and Alwan (1992) and Alwan and Alwan (1994) is to use multivariate control such as Hotelling's T^2 chart or multivariate CUSUM chart to treat observations of an autocorrelated univariate process. This is done by forming a multivariate vector of a moving window of observations from the process. In this approach it is necessary to choose the time delay between samples in such way that the constructed vectors are almost uncorrelated. Apley and Tsung (2002) modified this idea allowing correlation between samples.

The main purpose of this paper is to introduce an automatic and simpler form to monitor the process in the presence of correlation. The alternative we will propose does not depend upon the identification and adjustment of ARIMA models as well as the calculation of one step prediction errors. The idea is to use Geostatistics methodology (Cressie, 1993) to estimate the variance and the standard deviation of the process. The quality of the process is then monitored by the usual Shewhart's charts applied to original characteristic X of interest by replacing the classical standard deviation estimator in the Shewhart' control charts for a geostatistical estimator of σ. The correction of the charts due to presence of the correlation is automatically incorporated in the control limits UCL and LCL. The results of a simulation study comparing the geostatistical with the classical estimators will be presented.

Discussing the Effects of the Correlation: ARIMA Models

In the context of the ARIMA models it is interesting to observe that the correlation effect in the estimator s^2 is more accentuated in situations where the observations are generated by an auto regressive process (AR) where s^2 is the square of s in (2). To see this consider the AR(1) and MA(1) models defined as

$$X_t = \phi X_{t-1} + a_t + \delta \quad (5)$$

$$X_t = a_t - \theta a_{t-1} + \mu \quad (6)$$

where $|\phi| < 1$, $|\theta| < 1$, δ and μ are constants and $a_t \sim N(0, \sigma^2_a)$ is a white noise series. In the AR(1) and MA(1) models the first order autocorrelation is given by $\phi = \rho_1$ and $\rho_1 = \frac{-\theta}{1+\theta^2}$, respectively.

Zhang (1998) had shown that the expectation of the estimator s^2 for an autocorrelated process is given by

$$E\left[s^2\right] = \sigma^2\left[1 - \frac{2}{n(n-1)}\sum_{h=1}^{n-1}(n-h)\rho_h\right] \quad (7)$$

where $\rho_h = Corr(X_i, X_{i+h})$. As we can see from (7) if $\rho_h > 0$, $\forall h$, then $E[s^2]$ will be smaller than the true value s^2. If $\rho_h < 0$, $\forall h$, then $E[s^2]$ will be larger than σ^2. For processes with a mixture of positive and negative correlation $E[s^2]$ could be smalller or larger than the true value of σ^2 and for large sample sizes (7) converges to σ^2. For the AR(1) and MA(1) the expression (7) reduces respectively to:

$$E\left[s^2\right] = \sigma^2\left[1 - \frac{2}{n(n-1)}\phi\left(\frac{n - n\phi - 1 + \phi^n}{(1-\phi)^2}\right)\right] = C(n,\phi)\,\sigma^2 \quad (8)$$

$$E\left[s^2\right] = \sigma^2\left[1 + \frac{2\theta}{n(1+\theta^2)}\right] = C(n,\theta)\sigma^2 \quad (9)$$

Tables 1 and 2 show the values of $C(n, \phi)$ and $C(n, \theta)$ for samples of sizes $n = 25, 100$, $\phi \in [-0.9, 0.9]$ and $\theta \in [-0.9, 0.9]$. It can be seen that for AR(1) the bias of s^2 is higher for $n = 25$ and positive high correlation. For MA(1) model the bias is negligible for both sample sizes and for all values of θ.

Geostatistics Methodology

The Geostatistics methodology was initially formulated with the purpose to analyse geological data (Matheron, 1963). Nowdays, it has been used in many other fields. Several examples appear in the study of pluviometric precipitation or atmospheric data (Ord and Rees, 1979; Thiebaux and Pedder, 1987; Kitanidis,1997), or in study of ground water-flow (Cressie, 1993; Yeh et al.,1995). Geostatistics has also been applied for variables that are not of physical-chemistry nature such as rates of infantile mortality and abundance of species (Cressie, 1993). In quality control, applications of Geostatistics can be found in mining industry and in sampling of materials of continuous flow (Gy, 1998, 1982). Some

Table 1 – Values of C(n, ϕ) - AR(1).

ϕ	$n = 25$	$n = 100$
0.90	0.53	0.84
0.80	0.73	0.92
0.70	0.83	0.95
0.60	0.89	0.97
0.50	0.92	0.98
0.40	0.95	0.99
0.30	0.97	0.99
0.20	0.98	1.00
0.10	0.99	1.00
0.00	1.00	1.00
- 0.10	1.01	1.00
- 0.20	1.01	1.00
- 0.30	1.02	1.00
- 0.40	1.02	1.01
- 0.50	1.03	1.01
- 0.60	1.03	1.01
- 0.70	1.03	1.01
- 0.80	1.04	1.01
- 0.90	1.04	1.01

Table 2 – Values of $C(n, \phi)$ - MA(1).

θ	ρ_1	$n = 25$	$n = 100$
0.90	- 0.50	1.04	1.01
0.80	- 0.49	1.04	1.01
0.70	- 0.47	1.04	1.01
0.60	- 0.44	1.04	1.01
0.50	- 0.40	1.03	1.01
0.40	- 0.34	1.03	1.01
0.30	- 0.28	1.02	1.01
0.20	- 0.19	1.02	1.00
0.10	- 0.10	1.01	1.00
0.00	0.00	1.00	1.00
- 0.10	0.10	0.99	1.00
- 0.20	0.19	0.98	1.00
- 0.30	0.28	0.98	0.99
- 0.40	0.34	0.97	0.99
- 0.50	0.40	0.97	0.99
- 0.60	0.44	0.96	0.99
- 0.70	0.47	0.96	0.99
- 0.80	0.49	0.96	0.99
- 0.90	0.50	0.96	0.99

general references in Geostatistics are Cressie (1993), Journell and Huijbregts (1997), Chilès and Delfiner (1999) and Houlding (2000).

Briefly speaking, suppose we have a random sample of a random variable X collected in many different locations from a certain area. In this case, statistical models are build with the main objective to predict the value of X for locations not in the original sample. These models incorporate the information of the existing relationship among the sample values of X for different locations through a function called semivariogram (or variogram) which plays an important role in the spatial prediction procedure called Kriging (Cressie, 1993). In the Kriging procedure the value of X for a new location with coordinates s_0 for example, is predicted based upon the values of X in a neighborhood of s_0. Although Geostatistics can be used for locations in $\Re^d$ space most of the applications are related to $\Re^2$. Next we will introduce the Geostatistics definitions in $\Re$ space.

Geostatistics in the $\Re$ Domain: Main Concepts

The sequence of observed values of the quality characteristic X can be treated as a trajectory of a stochastic process $\{X(t), t \in \Re\}$.The variability of the process can be expressed in terms of the theoretical semivariogram of the process. Two assumptions are necessary: intrinsically stationarity and the isotropy. Shortly, these assumptions are described as follows:

A. Intrinsically Stationarity: The stochastic process $\{X(t), t \in \Re\}$ is such that:

(i) $E[X(t)] = \mu, \forall t \in \Re$;

(ii) $\mathrm{Var}[X(t_l) - X(t_k)] = 2\gamma(\|t_l - t_k\|), \forall t_l \neq t_k, \in \Re$,

which means that the process has constant average in $\Re$, and for all $t_l, t_k \in \Re$, $t_l \neq t_k$, the variance of the difference $[X(t_l) - X(t_k)]$ is a function only of the difference $\|t_l - t_k\|$ depending on its magnitude and direction. The funcions $2\gamma(\bullet)$ and $\gamma(\bullet)$ are called variogram and semivariogram of the process, respectively.

B. Isotropy: If the variogram $2\gamma(\bullet)$ is a function only of the distance among the sample units then the process is said to be isotropic.

In the case of industrial processes, condition i) is equivalent to say that the process is "under control" in relation to the average and condition ii) indicates that the variability of the difference between any two observations of the process is just a function of the distance between them. The isotropy means that the future and the past of the process are described by the same variogram function. In practice, the intrinsically stationarity and isotropy are reasonable assumptions for the industrial processes when they are in the "under control" condition. The $\Re$ space covers situations where samples were collected on time domain as well situations where samples were collected in some specific order not necessarily time. Therefore, each sample has its own "reference location" in the space and Geostatistics can be applied to analyse the data. Theoretically, it is expected that the correlation between any two sample units of the process decreases to zero as the distance between them increases. Therefore, after a certain point c the natural variability is the only source affecting the process. Some common semivariograms models are: spherical, linear, gaussian, exponential and wave (Cressie, 1993). In practice the theoretical semivariogram is estimated by using a sample of the process $\{X(t), t \in \Re\}$.

At this point it is interesting to notice that for intrinsically stationarity and isotropic processes the semivariogram $\gamma(\bullet)$ can be expressed as

$$\gamma(h) = 1/2\{Var[X(t+h) - X(t)]\} = 1/2\{Var[X(t+h)] + Var[X(t)]\} - Cov[X(t), X(t+h)] = \sigma^2 - \sigma^2 \, Corr[X(t), X(t+h)] = \sigma^2(1 - \rho_h), \ \forall\, h \tag{10}$$

where ρ_h is the correlation between X_i and X_j, $|i - j| = h$, $i \neq j$.When the correlation is equal to zero the semivariogram of order h is equal to the natural variance σ^2 of the process. By the Eq. (10) it is clear that in order to estimate the variance σ^2 it will be enough to have estimators of the semivariogram $\gamma(\bullet)$ and the correlation of order h, ρ_h. Therefore, it is possible to create many alternative estimators for the variance σ^2 that automatically will take into account the correlation of the process. There are many semivariogram estimators for $\gamma(h)$, called experimental semivariograms (Cressie, 1993) but Matheron's (1963) classic estimator is the most well known. Given a sample of n observations of the process, denoted by $X_1, X_2, \ldots, X_n$, Matheron's estimator of $\gamma(h)$ is defined as

$$\hat{\gamma}(h) = \frac{1}{2}\,\frac{\sum_{i=1}^{n-h}\left[X_i - X_{i+h}\right]^2}{n-h}, \ \forall\, h \in T \tag{11}$$

where X_i is the value of the quality characteristic X for the sample unit i, $i = 1, 2, \ldots, n$, $T = \{1, 2, \ldots, n-1\}$, $(n - h)$ is the number of pairs (X_i, X_j) such that $|i - j| = h$, $i \neq j$. The autocorrelation function of order h, ρ_h, is estimated by

$$\hat{\rho}_h = \frac{\sum_{i=1}^{n-h}\left(X_i - \overline{X}\right)\left(X_{i+h} - \overline{X}\right)}{\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2} \tag{12}$$

where $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ is the sample mean. The functions $(\gamma(h), \rho_h)$ are estimated under the assumption that the process is intrinsically stationary and isotropic which is the same as saying that the process is "under control". In the next section we present the new estimators for σ^2 that will be discussed in this paper.

New Estimators for the Process Variability: Geostatistics Approach

In Mingoti (2000) and Neves (2001) several estimators were proposed to estimate the variance σ^2 of autocorrelated processes based on Geostatistics methodology. In this paper we will present a comparison of the geoestatistical estimators with the classical estimators (2) and (3). They were constructed considering the relation (10) of section 3. All geostatistical estimators are biased but the bias converges to zero for large samples. In all cases an estimate of the standard deviation is obtained by taking the square root of the estimated variance.

The estimator $\hat{\sigma}^2_1$ defined as

$$\hat{\sigma}^2_1 = \frac{\hat{\gamma}_1}{1-\hat{\rho}_1} \quad (13)$$

takes into account only the semivariogram and autocorrelation of order 1 and it is very simple to calculate. The estimator $\hat{\sigma}^2_2$ defined as

$$\hat{\sigma}^2_2 = \frac{\sum_{h=1}^{3} \frac{\hat{\gamma}_h}{3}}{1-\sum_{h=1}^{3} \frac{\hat{\rho}_h}{3}} \quad (14)$$

takes into account the three first semivariogram and autocorrelation values. It is an option for process which have significant correlations of order higher than 1. The estimator $\hat{\sigma}^2_3$ introduced by Mingoti and Fidelis (2001) is the average of the M semivariogram values, where M is a constant in the set $\{1,2,\ldots,(n-1)\}$. In practice M should be chosen in the neigboorhood of [n/2], where [x] denotes the larger integer number less or equal to x, and such that the number of pairs (X_i, X_j) used to estimate $\gamma(h)$ is higher ou equal to 30. This is the region where the semivariogram is estimated with better precision (Journel and Huijbregts, 1997).

$$\hat{\sigma}^2_3 = \frac{1}{M}\sum_{h=1}^{M}\hat{\gamma}_h \quad (15)$$

The estimator $\hat{\sigma}^2_4$ is an extension of the estimator $\hat{\sigma}^2_3$ and the correction term in the denominator has the purpose to decrease the bias of the estimator $\hat{\sigma}^2_3$.

$$\hat{\sigma}^2_4 = \frac{\sum_{h=1}^{M}\hat{\gamma}_h}{\sum_{h=1}^{M}(1-\hat{\rho}_h)} \quad (16)$$

Finally, the estimator $\hat{\sigma}^2_5$ is a modification of $\hat{\sigma}^2_4$ where M is defined as in $\hat{\sigma}^2_3$. The purpose of using more than just 3 semivariogram values to estimate σ^2 is to increase the precision.

$$\hat{\sigma}^2_5 = \frac{1}{M}\sum_{h=1}^{M}\frac{\hat{\gamma}_h}{(1-\hat{\rho}_h)} \quad (17)$$

The nice thing about the geostatistical estimators defined in this section is that there is no need to recognize and adjust a statistical model to the sample series of the process or to the experimental process variogram something that would be necessary if one would decide to use the "best linear unbiased estimator" obtained by using Kriging technique. Usually in $\Re$ space the experimental variogram takes to a wave form and the estimation of its parameters is not very simple (see Mingoti and Neves, 1999 for details).

The geostatistical estimators can also be used in situations where the process is monitored by using averages of rational groups. If $\overline{X}_1, \overline{X}_2, \ldots, \overline{X}_m$ represent the average values of m sample groups then formulas (11) and (12) should be applied to the $\overline{X}_k$ values, $k = 1, 2, \ldots, m$, to estimate the semivariogram and correlation of order h of the theoretical stochastic process which is generating the $\overline{X}_k$ average values. The new formulas are defined as

$$\bar{\hat{\gamma}}(h) = \frac{1}{2}\,\frac{\sum_{i=1}^{m-h}\left[\overline{X}_i - \overline{X}_{i+h}\right]^2}{m-h}, \ \forall\, h \in T^* \tag{18}$$

$$\bar{\hat{\rho}}_h = \frac{\sum_{i=1}^{m-h}\left(\overline{X}_i - \overline{\overline{X}}\right)\left(\overline{X}_{i+h} - \overline{\overline{X}}\right)}{\sum_{i=1}^{n}\left(\overline{X}_i - \overline{\overline{X}}\right)^2} \tag{19}$$

where $\overline{\overline{X}} = \frac{1}{m}\sum_{i=1}^{m}\overline{X}_i$ is the global mean, $T^* = \{1, 2, \ldots, m-1\}$, and $(m-h)$ is the number of pairs $(\overline{X}_i, \overline{X}_j)$ such that $|i-j| = h$, $i \neq j$. The Shewhart's control limits for the average of the process are then given by

$$UCL = \overline{\overline{X}} + k\,\hat{\sigma}_i;\ CL = \overline{\overline{X}};\ LCL = \overline{\overline{X}} - k\,\hat{\sigma}_i \tag{20}$$

where $\hat{\sigma}_i$, is any geostatistical estimator presented in this section calculated with the semivariogram and correlation estimates given by Eqs. (18) and (19), $i = 1, 2, 3, 4, 5$.

Simulation Results

In this section we present the results of a Monte Carlo simulation study performed to evaluate the geostatistical estimators presented in section 4 for the standard deviation of the process. They were also compared to the classical estimator s and the moving sample range $\hat{\sigma}_{AM}$ defined in section 1. Samples with sizes $n = 25$, 50 and 100 were generated from an AR(1) with parameter $\phi \in [-0.9, 0.9]$ and from an ARMA(1,1) with the parameters (ϕ, θ) chosen such that $\rho_1 \in [-0.95, 0.95]$. The region of simulation contains models with long and short, positive and negative, correlation structure. As an illustration, for the AR(1) with $\phi = 0.9$ the autocorrelation for $h = 7$ is equal to 0.478 while for the AR(1) with $\phi = 0.7$ the autocorrelation for $h = 3$ is only 0.34. The choice of AR(1) and ARMA(1,1) was due to the fact that those are the more commom models for autocorrelated processes according to the literature (see Box and Luceno, 1997; Zhang, 1998). All generated processes had the same fixed mean. The white noise was generated from a normal distribution with zero mean and standard deviation σ_a ranging from 2 to 7. The constant $M = [n / 2]$ was used for the geostatistical estimators when needed. A total of $r = 100$ samples were generated for each case and the Mean Error (*ME*), the Mean Absolute Error (*MAE*) and the Mean Square Error (*MSE*), were calculated for all the estimators. The average results for the AR(1) and ARMA (1,1) considering all simulated cases, are shown in Tables 3 and 4 as a function of the true correlation ρ_1. The respective values of (ϕ, θ) are also shown in the Table 4. Table 5 presents the average results as a function of the white noise standard deviation σ_a. From Tables 3 and 4 it can be seen that in the presence of correlation the geostatistical estimators had better or similar performance than the classical estimators s and $\hat{\sigma}_{AM}$. Among the geostatistical estimators in general, for high negative correlation the estimator $\hat{\sigma}_5$ had a better performance; for intermediate negative correlation $\hat{\sigma}_1$ and $\hat{\sigma}_2$ had smaller error values (*ME, MAE, MSE*) and for high positive correlation the estimators $\hat{\sigma}_1$ and $\hat{\sigma}_2$ had a better performance. When the correlation is small the average errors of all the estimators are more similar. For high positive correlation the estimator $\hat{\sigma}_3$ presented smaller errors than the classical estimators. Considering that the estimator $\hat{\sigma}_3$ does not have any correcting factor for bias this is an interesting result. In general the estimator $\hat{\sigma}_4$ presented larger errors than the estimator s but the difference was not very accentuated. In all cases the moving sample range estimator $\hat{\sigma}_{AM}$ had a very bad performance with larger error values especially the *MSE*. From Table 5 it can be seen that for all the estimators the error values increase as σ_a increases. The increase rate is much higher for the ARMA than the AR process. The geostatistical estimator $\hat{\sigma}_5$ had superior performance for the ARMA process and a good performance for the AR. Table 6 presents the average results as a function of the sample size n. As expected for all the estimators the errors decrease as n increases. In general the errors (*ME, MAE, MSE*) are larger for ARMA than for AR process. The *MSE* values for the $\hat{\sigma}_{AM}$ are intolerable for small and larger sample sizes. By considering the average results *ME, MAE, MSE* for all cases presented in Tables 3 and 4 for AR(1) and ARMA(1,1) we can see that the classical standard sample deviation had smaller values only in 2 cases for AR(1) and in 4 cases for ARMA(1,1) compared to the geostatistical estimators.

Example of Application

Table 7 presents the observed values of waiting time in line (in minutes) for 40 customers of a laboratory. The autocorrelation and semivariogram estimates for $h = 1, 2, \ldots, 20$, are presented in Table 8. Table 9 shows the obtained estimates for the standard deviation σ using all 7 estimators discussed in this paper. As an example, Shewhart's control charts for the average waiting time using the sample standard deviation s and the geostatistical estimator $\hat{\sigma}_1$ are presented in Figure 1. As one can see the control limits calculated by using the estimator $\hat{\sigma}_1$ are shorter than the limits calculated using the sample standard deviation. The moving range estimate was the smallest value and completelly different from all the others 6 estimators (see Table 9).

Table 3 – Average results for the geostatistical and classical estimators of the standard deviation of the process - AR(1).

$\|\rho_1\|$		$\rho_1 > 0$			$\rho_1 < 0$		
	$\hat{\sigma}_i$	*ME*	*MAE*	*MSE*	*ME*	*MAE*	*MSE*
	1	- 0.4243	1.0335	1.9583	0.5342	1.1923	3.0837
	2	- 0.3809	1.0602	2.0534	0.5670	1.1920	3.0572
	3	- 0.0031	1.2224	2.8055	0.6467	1.2462	3.2508
0.90	4	0.4943	1.2181	2.9182	0.5268	1.1978	2.9763
	5	0.2564	1.1317	2.4271	0.2600	1.1031	2.3821
	6	- 7.2270	7.2270	59.7367	5.3278	5.3344	40.3307
	7	0.4480	1.2065	2.8856	0.5719	1.2141	3.2336
	1	- 0.2413	0.7720	1.1740	0.2077	0.8351	1.4732
	2	- 0.2185	0.7871	1.2068	0.2201	0.8325	1.4526
	3	0.0899	0.9275	1.6343	0.2881	0.8836	1.6062
0.85	4	0.3130	0.8935	1.5355	0.1893	0.8570	1.4949
	5	0.1798	0.8500	1.3690	0.0549	0.8345	1.3871
	6	- 5.2959	5.2959	32.2813	3.7714	3.7858	19.8403
	7	0.2457	0.8516	1.4333	0.2415	0.8503	1.5427
	1	- 0.2010	0.6825	0.8956	0.0958	0.6773	0.8712
	2	- 0.1619	0.6797	0.9050	0.0960	0.6727	0.8605
	3	0.0703	0.7961	1.2115	0.1511	0.7020	0.9484
0.80	4	0.1812	0.7422	1.0662	0.0646	0.6847	0.8909
	5	0.0982	0.7189	0.9950	- 0.0219	0.6808	0.8742
	6	- 4.2090	4.2090	20.4759	2.9620	2.9677	12.0537
	7	0.2149	0.9275	1.5323	0.2269	0.9271	1.5205
	1	- 0.1799	0.6220	0.7267	0.0318	0.5992	0.7082
	2	- 0.1431	0.6285	0.7530	0.0268	0.5989	0.7045
	3	0.0581	0.6992	0.9559	0.0847	0.6213	0.7661
0.75	4	0.1092	0.6598	0.8585	0.0075	0.6093	0.7355
	5	0.0480	0.6456	0.8118	- 0.0530	0.6069	0.7315
	6	- 3.4686	3.4686	13.9468	2.4578	2.4793	8.5000
	7	0.0695	0.6277	0.7727	0.0649	0.6064	0.7284
	1	- 0.1604	0.5613	0.6068	0.0069	0.5528	0.5689
	2	- 0.1314	0.5676	0.6139	- 0.0022	0.5513	0.5638
	3	0.0195	0.6216	0.7572	0.0585	0.5734	0.6147
0.70	4	0.0521	0.5877	0.6726	- 0.0127	0.5656	0.5978
	5	0.0066	0.5803	0.6491	- 0.0584	0.5659	0.5989
	6	- 2.9176	2.9176	9.9615	2.0835	2.1026	6.1243
	7	0.0444	0.5645	0.6294	0.0412	0.5590	0.5847
	1	- 0.1536	0.4936	0.4826	- 0.0161	0.4821	0.4682
	2	- 0.1241	0.4918	0.4807	- 0.0290	0.4831	0.4719
	3	- 0.0074	0.5239	0.5592	0.0282	0.5026	0.5105
0.60	4	- 0.0109	0.4990	0.4994	- 0.0332	0.4969	0.4953
	5	- 0.0399	0.4972	0.4931	- 0.0631	0.4984	0.4972
	6	- 2.1700	2.1700	5.5370	1.5931	1.6205	3.8033
	7	- 0.0100	0.4831	0.4669	0.0195	0.4871	0.4766
	1	- 0.1540	0.4533	0.3955	- 0.0533	0.4318	0.3618
	2	- 0.1301	0.4529	0.3933	- 0.0670	0.4340	0.3640
	3	- 0.0416	0.4660	0.4170	- 0.0141	0.4480	0.3868
0.50	4	- 0.0605	0.4539	0.3929	- 0.0695	0.4461	0.3812
	5	- 0.0825	0.4540	0.3922	- 0.0920	0.4479	0.3831
	6	- 1.6044	1.6047	3.1773	1.1886	1.2228	2.2513
	7	0.0069	0.4857	0.4554	0.0214	0.4877	0.4586
	1	- 0.0823	0.4021	0.3111	- 0.0563	0.4130	0.3420
	2	- 0.0660	0.4011	0.3073	- 0.0651	0.4137	0.3425
	3	- 0.0024	0.4054	0.3142	- 0.0143	0.4222	0.3527
0.30	4	- 0.0343	0.4024	0.3086	- 0.0613	0.4195	0.3510
	5	- 0.0490	0.4027	0.3081	- 0.0757	0.4199	0.3526
	6	- 0.8069	0.8303	0.9973	0.6464	0.7580	0.9911
	7	- 0.0024	0.3974	0.3032	- 0.0141	0.4156	0.3456

(*) $\hat{\sigma}_6 = \hat{\sigma}_{AM}$ is the classical moving range estimator;
$\hat{\sigma}_7 = s$ is the classical standard sample deviation.

Volume 2, Number 1, 2005, pp. 39-56

Table 4 – Average results for the geostatistical and classical estimators of the standard deviation of the process - ARMA(1,1).

	(ρ_1, ϕ, θ)											
	(- 0.95, - 0.9, 0.9)			(- 0.88, - 0.9, - 0.1)			(- 0.71, - 0.5, 0.5)			(- 0.64, - 0.7, - 0.1)		
$\hat{\sigma}_i$	*ME*	*MAE*	*MSE*	*ME*	*MAE*	*MSE*	*ME*	*MAE*	*MSE*	*ME*	*MAE*	*MSE*
1	1.7894	3.2760	23.0016	0.4372	1.0220	2.1037	- 0.0746	0.5112	0.4993	- 0.0987	0.4085	0.3195
2	1.9032	3.3174	23.5941	0.4613	1.0242	2.0877	- 0.0850	0.5123	0.4980	- 0.1085	0.4097	0.3202
3	2.0993	3.4337	25.0951	0.5513	1.0969	2.3307	- 0.0410	0.5222	0.5161	- 0.0626	0.4127	0.3257
4	1.8692	3.3286	23.4263	0.4424	1.0540	2.1274	- 0.1049	0.5236	0.5166	- 0.1060	0.4157	0.3284
5	1.2222	3.0179	17.8430	0.2196	0.9780	1.7826	- 0.1413	0.5282	0.5252	- 0.1203	0.4176	0.3303
6	11.5237	11.5452	203.9942	4.7097	4.7179	30.7231	1.7022	1.7260	4.3521	0.4791	0.6207	0.7237
7	1.8379	3.3047	23.5058	0.4738	1.0413	2.2114	- 0.0404	0.5130	0.5064	- 0.0518	0.4059	0.3180
	(ρ_1, ϕ, θ)											
	(- 0.47, - 0.1, 0.5)			(- 0.24, - 0.7, - 0.5)			(0.95, 0.9, - 0.9)			(0.88, 0.9, 0.1)		
$\hat{\sigma}_i$	*ME*	*MAE*	*MSE*	*ME*	*MAE*	*MSE*	*ME*	*MAE*	*MSE*	*ME*	*MAE*	*MSE*
1	0.0545	0.6276	0.7691	0.0028	0.4633	0.4202	- 0.5683	1.4507	5.4771	- 0.3584	0.9148	1.5531
2	0.0386	0.6293	0.7717	- 0.0109	0.4647	0.4233	- 0.5293	2.4852	10.8687	- 0.3661	0.9266	1.6054
3	0.1040	0.6611	0.8750	0.0426	0.4803	0.4502	0.0894	2.7353	13.6426	0.0107	1.0844	2.1658
4	0.0237	0.6483	0.8293	- 0.0160	0.4746	0.4340	1.1699	2.8752	16.1518	0.4046	1.0514	2.0852
5	- 0.0196	0.6466	0.8209	- 0.0373	0.4749	0.4331	0.6720	2.6774	13.5529	0.2025	0.9723	1.7746
6	2.3295	2.3449	7.7694	1.2061	1.2420	2.3625	- 14.7894	14.7894	250.7075	- 6.2217	6.2217	44.4910
7	0.0894	0.6351	0.7888	0.0428	0.4682	0.4286	1.0407	2.8365	15.9467	0.3543	1.0177	2.0021
	(ρ_1, ϕ, θ)											
	(0.71, 0.5, - 0.5)			(0.64, 0.7, 0.1)			(0.47, 0.1, - 0.5)			(0.24, 0.7, 0.5)		
$\hat{\sigma}_i$	*ME*	*MAE*	*MSE*	*ME*	*MAE*	*MSE*	*ME*	*MAE*	*MSE*	*ME*	*MAE*	*MSE*
1	- 0.1944	0.5284	0.5304	- 0.1093	0.5729	0.5704	- 0.1764	0.6125	0.6839	- 0.0875	0.4705	0.4322
2	- 0.1763	0.5216	0.5177	- 0.1340	0.4056	0.3079	- 0.0950	0.6110	0.6798	- 0.0447	0.4719	0.4432
3	- 0.0458	0.5493	0.5850	- 0.0687	0.4018	0.3098	0.0456	0.6460	0.7878	0.0225	0.4835	0.4842
4	- 0.0209	0.5272	0.5375	- 0.0899	0.4007	0.3066	0.0508	0.6186	0.7137	- 0.0049	0.4736	0.4579
5	- 0.0578	0.5231	0.5260	- 0.1031	0.4022	0.3077	0.0077	0.6110	0.6919	- 0.0254	0.4731	0.4538
6	- 2.4467	2.4467	7.0452	- 0.6859	0.7212	0.7715	- 3.2044	3.2044	11.9181	- 1.4797	1.4820	2.7502
7	- 0.0248	0.5084	0.5020	- 0.0585	0.3963	0.3019	0.0509	0.6040	0.6835	0.0284	0.4676	0.4374

(*) $\hat{\sigma}_6 = \hat{\sigma}_{AM}$ is the classical moving range estimator; $\hat{\sigma}_7 = s$ is the classical standard sample deviation.

Table 5 – Average results for the geostatistical and classical estimators of the standard deviation of the process as a function of σ_a.

		AR			ARMA		
σ_a	$\hat{\sigma}_i$	*ME*	*MAE*	*MSE*	*ME*	*MAE*	*MSE*
2	1	- 0.0237	0.2832	0.1520	0.0224	0.5329	0.7665
	2	- 0.0177	0.2845	0.1540	0.0553	0.5325	0.7763
	3	0.0361	0.3054	0.1774	0.1428	0.5638	0.8733
	4	0.0429	0.2946	0.1657	0.1967	0.5583	0.8735
	5	0.0076	0.2876	0.1530	0.1053	0.5245	0.7253
	6	- 0.2145	1.3358	2.5749	- 0.3144	2.3895	10.5233
	7	0.0509	0.2868	0.1636	0.1935	0.5526	0.8580
3	1	- 0.0229	0.4152	0.3316	0.0311	0.7627	1.4979
	2	- 0.0112	0.4161	0.3346	0.0795	0.7647	1.5192
	3	0.0690	0.4535	0.3983	0.2009	0.8142	1.7217
	4	0.0795	0.4400	0.3763	0.2810	0.8119	1.7383
	5	0.0264	0.4286	0.3424	0.1482	0.7591	1.4371
	6	- 0.3150	1.9818	5.7008	- 0.5067	3.5429	22.9923
	7	0.0909	0.4284	0.3640	0.2858	0.8046	1.7475
4	1	- 0.0356	0.5660	0.6506	0.0555	1.0526	3.0655
	2	- 0.0221	0.5702	0.6629	0.1239	1.0576	3.1420
	3	0.0912	0.6156	0.7784	0.2964	1.1361	3.6183
	4	0.1086	0.5962	0.7352	0.4058	1.1232	3.5922
	5	0.0372	0.5782	0.6633	0.2243	1.0511	2.9407
	6	- 0.4496	2.6911	10.7485	- 0.5989	4.8039	43.1554
	7	0.1251	0.5842	0.7263	0.4028	1.1021	3.5077
5	1	- 0.0667	0.7098	0.9581	0.0264	1.2580	4.1816
	2	- 0.0538	0.7117	0.9606	0.1082	1.2630	4.2568
	3	0.0796	0.7589	1.0925	0.3220	1.3619	4.9229
	4	0.0964	0.7338	1.0247	0.4554	1.3399	4.8579
	5	0.0079	0.7166	0.9442	0.2354	1.2588	4.0141
	6	- 0.5139	3.3379	16.1486	- 0.8598	5.8936	63.5967
	7	0.1210	0.7226	1.0271	0.4447	1.3055	4.7462
6	1	- 0.0905	0.8391	1.3514	0.0747	1.5422	6.1699
	2	- 0.0665	0.8444	1.3676	0.1789	1.5454	6.2524
	3	0.1058	0.9154	1.6141	0.4219	1.6432	7.0560
	4	0.1230	0.8868	1.5326	0.5889	1.6352	7.1192
	5	0.0170	0.8667	1.4040	0.3151	1.5236	5.8177
	6	- 0.6564	3.9639	22.7816	- 0.9438	7.1322	92.6429
	7	0.1372	0.8641	1.4957	0.5983	1.6235	7.1873
7	1	- 0.0779	1.0132	1.9667	0.0041	1.8041	8.7616
	2	- 0.0573	1.0157	1.9691	0.1235	1.8092	8.8877
	3	0.1479	1.0991	2.3484	0.4312	1.9319	10.0389
	4	0.1704	1.0734	2.2310	0.6127	1.9265	10.1370
	5	0.0420	1.0364	1.9878	0.2977	1.8090	8.2329
	6	- 0.7264	4.6872	32.0489	- 1.2109	8.2380	125.4821
	7	0.1929	1.0529	2.2022	0.5953	1.8946	10.1568

(*) $\hat{\sigma}_6 = \hat{\sigma}_{AM}$ is the classical moving range estimator;
$\hat{\sigma}_7 = s$ is the classical standard sample deviation.

Table 6 – Average results for the geostatistical and classical estimators of the standard deviation of the process as a function of *n* (positive correlation).

		AR (1)			ARMA (1,1)		
n	$\hat{\sigma}_i$	ME	MAE	MSE	ME	MAE	MSE
25	1	- 0.1047	0.8810	1.6116	0.0624	1.4900	6.3739
	2	- 0.0777	0.8863	1.6219	0.1732	1.5023	6.5316
	3	0.1038	0.9381	1.8308	0.3697	1.5676	7.0518
	4	0.1352	0.9040	1.7246	0.5916	1.5669	7.1198
	5	0.0002	0.8722	1.5255	0.2512	1.4274	5.4412
	6	- 0.3946	3.1806	17.5410	- 0.3650	5.7222	70.7773
	7	0.1961	0.9107	1.8032	0.6759	1.5886	7.4756
50	1	- 0.0389	0.6033	0.7235	0.0469	1.1244	3.7361
	2	- 0.0277	0.6058	0.7311	0.1222	1.1281	3.7905
	3	0.1004	0.6628	0.9067	0.3509	1.2284	4.5496
	4	0.1098	0.6456	0.8600	0.4490	1.2188	4.5964
	5	0.0396	0.6289	0.7938	0.2705	1.1511	3.8849
	6	- 0.4914	2.9513	14.3296	- 0.7816	5.2984	58.1726
	7	0.1076	0.6223	0.7957	0.4090	1.1853	4.4118
100	1	- 0.0150	0.4290	0.3702	- 0.0022	0.8619	2.1116
	2	- 0.0089	0.4292	0.3715	0.0393	0.8559	2.0950
	3	0.0606	0.4731	0.4671	0.1870	0.9295	2.5142
	4	0.0654	0.4629	0.4482	0.2296	0.9118	2.4430
	5	0.0293	0.4560	0.4281	0.1413	0.8846	2.2578
	6	- 0.5520	2.8670	13.1310	- 1.0707	4.9793	50.2466
	7	0.0552	0.4365	0.3906	0.1753	0.8676	2.2144

(*) $\hat{\sigma}_6 = \hat{\sigma}_{AM}$ is the classical moving range estimator;
$\hat{\sigma}_7 = s$ is the classical standard sample deviation.

Table 7 – Customers waiting time data.

Customer	Waiting time	Customer	Waiting time
1	5.60	21	10.44
2	6.94	22	11.37
3	7.85	23	10.52
4	5.10	24	8.44
5	6.40	25	10.93
6	9.00	26	12.79
7	7.70	27	11.38
8	9.96	28	10.59
9	8.82	29	9.12
10	5.04	30	7.18
11	7.25	31	5.84
12	10.32	32	6.27
13	10.16	33	8.99
14	9.20	34	10.96
15	9.70	35	11.18
16	9.05	36	11.80
17	9.27	37	10.61
18	10.20	38	10.21
19	11.96	39	7.67
20	11.13	40	5.82

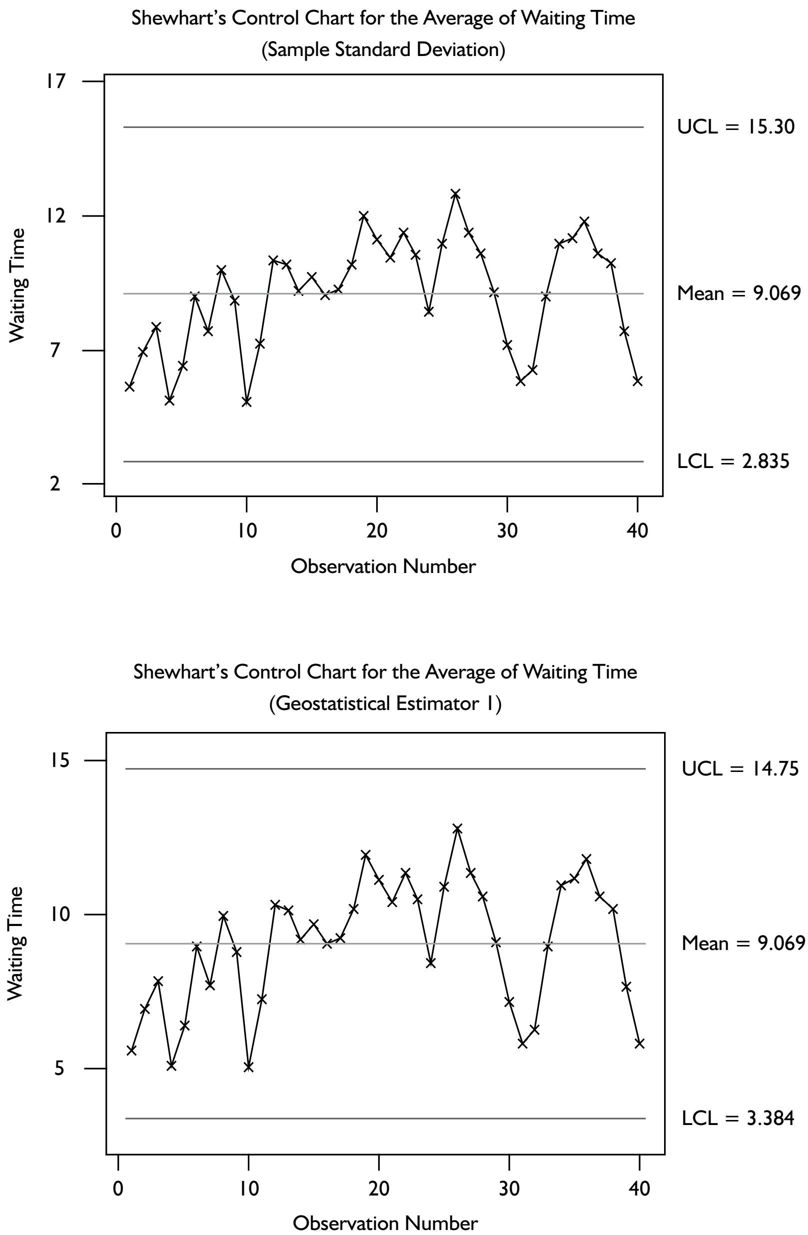

Figure 1 – Shewhart's control charts for the average of customers waiting time.

Volume 2, Number 1, 2005, pp. 39-56

Table 8 – Semivariogram and autocorrelation estimates waiting time - queuing system example.

h	$\hat{\rho}h$	$\hat{\gamma}h$	h	$\hat{\rho}h$	$\hat{\gamma}h$
1	0.6024	1.4279	11	4.4173	1.4279
2	0.2152	3.0966	12	4.9501	3.0966
3	0.1187	3.5818	13	4.6301	3.5818
4	- 0.0861	4.3997	14	4.5139	4.3997
5	- 0.1082	4.4114	15	4.6467	4.4114
6	0.0909	3.4886	16	4.9387	3.4886
7	0.1777	3.0692	17	5.5341	3.0692
8	0.1958	3.0572	18	6.1187	3.0572
9	0.1612	3.2162	19	5.3455	3.2162
10	0.0388	3.5668	20	4.8959	3.5668

Table 9 – Estimates of the standard deviation waiting time example.

Estimator	**Estimate**
Geostatistics 1	1.8950
Geostatistics 2	1.9819
Geostatistics 3	2.0409
Geostatistics 4	2.0433
Geostatistics 5	2.0206
Moving range	1.2513
Sample standard deviation	2.0783

Concluding Remarks

In this paper we presented new estimators for the variance and standard deviation of autocorrelated processes based upon the concepts of Geostatistics methodology. In the presence of correlation this estimation procedure is very appealing because it allows the user to keep monitoring the quality of the process by using the usual Shewhart's control charts. It was shown that in general the geostatistical estimators $\hat{\sigma}_1$ and $\hat{\sigma}_2$ had better or similar performance than the classical standard sample deviation s in all simulated cases. In the cases where the classical standard sample deviation s presents better performance than the geostatistical estimators $\hat{\sigma}_3$, $\hat{\sigma}_4$, $\hat{\sigma}_5$, the difference in terms of average error values were not to large. For high negative correlation the estimator $\hat{\sigma}_5$ was the best and for all the other cases the estimators $\hat{\sigma}_1$ and $\hat{\sigma}_2$ had better performance. This paper also shows that the classical moving sample range estimator should not be used to estimate the standard deviation of autocorrelated processes. This fact was also pointed out by Mingoti and Neves (2003).

Acknowlegement

This work was partially supported by the Brazilian Institution CNPq.

References

Alwan, A. J., Alwan, L. C. (1994) "Monitoring Autocorrelated Processes Using Multivariate Quality Control Charts", Proceedings of the Decision Sciences Institute Annual Meeting, 3, pp. 2106-2108.

Alwan, L. C., Roberts, H. V. (1995) "The Problem of Missplaced Control Limits", Applied Statistics, JRSS, Series C, Vol 44, No 3, pp. 269-278.

Alwan, L. C., Roberts, H. V. (1989) "Time Series Modelling for Statistical Process Control", Journal of Business & Economic Statistics, Vol 6, No 1, pp.87-95.

Apley, D.W., Tsung, F. (2002) " The Autoregressive T^2 Chart for Monitoring Univariate Autocorrelated Processes", Journal of Quality Technology, Vol 34, No 1, pp. 80-96.

Box, G.P., Jenkins, G.M. (1976), Time Series Analysis: Forecasting and Control, Holden Day, New York.

Box, G. P. Luceno, A. (1997), Statistical Control by Monitoring and Feedback Adjustment. John Wiley & Sons, New York.

Chilès, J. P., Delfiner, P. (1999), Geostatistics: Modelling Spatial Uncertainty, John Wiley & Sons, New York.

Cressie, N. (1993), Statistics for Spatial Data, John Wiley & Sons, New York.

Crowder, S. V. (1989) "Design of Exponentially Weighted Moving Average Schemes", Journal of Quality Technology, Vol 21, No 3, pp.155-162.

Gy, P. (1998), Sampling for Analytical Purposes, John Wiley & Sons, New York.

Gy, P. (1982), Sampling of Particulate Materials, Elsevier Scientific Publishers, New York.

Houlding,S. W. (2000), Practical Geostatistics, Springer Verlag, New York.

Hunter, J. S. (1998) "The Box-Jenkins Manual Bounded Adjustment Chart: Graphical Tool Designed for Use on the Production Floor", Quality Progress, Vol 31, No 8, pp.129-137.

Hunter, J. S. (1986) "The Exponentially Weighted Moving Average", Journal of Quality Technology, Vol 18, No 4, pp. 202-209.

Journel, A. G., Huijbregts, Ch. J. (1997), Mining Geostatistics, Academic Press, New York.

Kitanidis, P. K. (1997), Introduction to Geostatistics: Applications to Hydrogeology, Cambridge University Press, Cambridge.

Krieger, C. A., Champ, C. W., Alwan, L. C. (1992) " Monitoring an Autocorrelated Process", Presented at the Pittisburgh Conference on Modelling and Simulation, Pittisburgh, PA.

Lucas, J. M., Caccucci, M. S. (1990) "Exponentially Weighted Moving Average Control Schemes: Properties and Enhancements (with discussion)", Technometrics, Vol 32, No 1, pp. 1-129.

Matheron, G. (1963) "Principles of Geostatistics", Economic Geology Vol 58 No 8, pp. 1246-1266.

Mingoti, S. A. (2000), Aplicação de Novas Ferramentas Estatísticas no Monitoramento do Controle de Qualidade de Processos de Produção, Technical Report: CNPq (In Portuguese).

Mingoti, S. A., Neves, O. F. (1999) "A Metodologia de Geoestatística como Alternativa na Análise de Séries Temporais", Revista Escola de Minas, Vol 52, No.3, pp.182-187 (In Portuguese).

Mingoti, S. A., Fidelis, M. T. (2001) "Aplicando a Geoestatística no Controle Estatístico de Processos", Revista Produto & Produção, Vol 5, No 2, pp. 55-70 (In Portuguese).

Mingoti, S. A., Neves, O. F. (2003) "A Note on the Zhang Omnibus Test for Normality Based on the Q Statistics", Journal of Applied Statistics Vol 30, No 3, pp. 335-341.

Montgomery, D. C. (2001), Introduction to Statistical Quality Control, John Wiley & Sons, New York.

Montgomery, D. C, Mastrangelo, C. M. (1991) "Some Statistical Process Control Methods for Autocorrelated Data", Journal of Quality Technology, Vol 23, No 3, pp.179-193.

Neves, O. F. (2001), Estudo de Novos Estimadores para a Variabilidade de Processos. Master Dissertation. Statistics Department of Federal University of Minas Gerais, Belo Horizonte, Brazil. (In Portuguese).

Ord, J. K., Rees, M. (1979) "Spatial Processes: Recent Developments with Applications to Hydrology", in Mathematics of Hydrology and Water Resources, Academic Press, London, pp. 95-118.

Roberts, S. W. (1959) "Control Chart Tests Based on Geometric Moving Averages", Technometrics, Vol 1, No 3, pp. 239-251.

Runger, G. C., T. R. Willemain, T. R. (1995) "Model-Based and Model-Free Control of Autocorrelated Processes", Journal of Quality Technology, Vol 27, No 4, pp. 283-292.

Thiebaux, H. J., M. A. Pedder, M. A. (1987), Spatial Objective Analysis with Applications in Atmosphere Science, Academic Press, London.

Zhang, N. F. (1998) "Estimating Process Capability Indexes for Autocorrelated Data, Journal of Applied Statistics, Vol 25, No 4, pp. 559-574.

Yeah, T. C. J., Gutjahr, A. L., Jin, M. (1995) " An Interative Cokriging-Like Technique for Ground- Water Flow Modelling", Ground Water, Vol 33, No 1, pp. 33-41.

Biography

Sueli Mingoti holds a Ph.D. in Statistics from Iowa State University, Ames, Iowa, USA in 1989 and she is currently an Adjunct Professor at the Department of Statistics of UFMG (Federal University of Minas Gerais), Belo Horizonte, Brazil

Otaviano Francisco Neves is bachelor in Statistics by Federal University of Minas Gerais and holds a Master degree in Statistics from Federal University of Minas Gerais, Belo Horizonte, Brazil since 2001.