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Economic-Statistical Control Chart Design: A Sensitivity Study

Maysa S. de Magalhães
Escola Nacional de Ciências Estatísticas (ENCE)
Instituto Brasileiro de Geografia e Estatística (IBGE)
Rio de Janeiro, RJ, Brazil
E-mail: maysa@ibge.gov.br

Antonio F.B. Costa
Faculdade de Engenharia de Guaratinguetá (FEG)
Universidade do Estado de São Paulo (UNESP)
Guaratinguetá, SP, Brazil
E-mail: fbranco@feg.unesp.br

Abstract
When an economic-statistical model for a control chart is considered the effect of the choice of the bounds on the average times until a signal on the cost for controlling the process, including the cost with non-conformities produced, and on the design parameters is a natural issue that arises. To have an idea of how the costs and the chart's parameters are affected by these bounds is an important guidance for the design of the control charts, that is, for the choice of the size of the samples, the intervals between samplings, and the factors used in determining the width of the control limits. A sensitivity analysis of the choice of these bounds on the cost and the design parameters is presented to the adaptive $\overline{X}$ chart.

Keywords: adaptive control charts, economic-statistical design, sensitivity analysis

Introduction

The design of control charts with respect to economic criteria has been a subject of interest during the last four decades.

Duncan (1956) first proposed the economic design of $\overline{X}$ control charts. Since then, various models have been proposed for a number of Shewhart-type chart. Literature surveys of related work are presented in Gibra (1975), Montgomery (1980), Vance (1983), and Ho and Case (1994).

One concern about economic design of control charts is that it focuses only on costs, but ignores statistical properties, and thus it is entirely possible to produce designs that are optimal in an economic sense but which may have very poor statistical performance (see, Woodall, 1986).

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Saniga (1989) proposed the economic-statistical design of control charts. His paper became the foundation in this area, and several authors have followed up on his work. McWilliams (1994) provided a Fortran program that enables user to determine economic, statistical or economic-statistical $\overline{X}$ chart designs. Saniga et al.(1995) presented a computer program for determining the economic-statistical design of a fraction defective or defects per unit chart. Montgomery et al.(1995) presented a paper on the economic-statistical design of the EWMA control chart, and a computer program for the paper is developed by Torang et al. (1995). All these papers are related to economic-statistical designs of control charts with fixed design parameters.

Many researchers have been working on the economic and the economic-statistical design of fixed parameters control charts (see, for example, Saniga and Montgomery, 1981; Rahim, 1989; Costa, 1993; Rahim and Costa, 2000; and the references therein).

In economic and economic-statistical designs, a cost fucntion is formulated taking into account the overall cost of controlling the quality of a process through a control chart. This function provides a device for selection of the design parameters, and for comparison between charts. In economic-statistical design, moreover, statistical constraints are imposed on the cost function, such as requiring a short average time for the control chart to signal a process deterioration or a long average time between false alarms.

In recent years, a new class of control charts has been proposed where the design parameters (sample size, sample interval and control limit coefficient) are allowed to change in an adaptive way, that is, one or more design parameters vary as a function of the process data. These charts are called adaptive control charts and they have proved to be more efficient than fixed parameters control charts in detecting small to moderate shifts in the process parameter being controlled.

While the statistical design of adaptive charts has been studied extensively, very little work has been done on economic and economic-statistical design of these charts.

Economic designs for variable sample size (VSS) $\overline{X}$ charts were studied by Flaig (1991) and Park and Reynolds (1994). Variable sampling interval (VSI) control charts were considered by Das et al. (1997), Das and Gosavi (1997), Bai and Lee (1998). Subsequently, Park and Reynolds (1999) developed an economic model for a variable sample size and sampling interval (VSSI) $\overline{X}$ control chart. Finally, De Magalhães et al. (2001) investigated the economic design of variable parameters (VP) charts, in which all design parameters are considered variable.

Prabhu et al. (1997) proposed an economic-statistical design for a VSSI $\overline{X}$ chart and De Magalhães et al. (2002) developed an economic-ststistical model for a VP $\overline{X}$ chart.

In this paper, we consider the economic-statistical model for the adaptive $\overline{X}$ control chart developed by De Magalhães et al. (2002).

Considering the proposed model, a sensitivity analysis of the effect of the choice of the bounds on the average time until a signal, when the process is in control and out of control, on the cost and the design parameters for the adaptive $\overline{X}$ control chart is presented.

There are three numbers that should be taken care of when designing a chart using an economic-statistical criterion: the overall cost, the average time until a signal, when the process is in control and out of control. The choice of the design parameters should be guided, qualitatively, by the following rationale: the overall cost and the average time to signal when the process is out of control should be minimized and the average time to signal when the process is in control should be maximized. Since these requirements might be incompatible, one is led to the sensitivity analysis in order to be able to arrive at a compromise.

To present the sensitivity analysis we need to introduce the VP $\overline{X}$ chart and the assumptions about the production process as well to furnish the expression of the expected cost per time unit.

Adaptive $\overline{\mathbf{X}}$ Control Chart

Suppose that an $\overline{X}$ control chart having all design parameters varying adaptively is employed to monitor a process whose quality characteristic of interest (say, X) is normally distributed with mean μ and variance σ^2. The target value of the process mean is represented by μ_0. The process is in control when $\mu = \mu_0$ and out of control when $\mu = \mu_0 + \delta\sigma$.

To utilize a control chart the user should specify: the sample size (n), the sampling interval (h) and the coefficient values used in determining the width of the control limits (k). These parameters are the design parameters of a control chart.

In a fixed parameter (FP) $\overline{X}$ chart, the design parameters are kept fixed during the production process. The VP $\overline{X}$ control chart is a modification of the FP $\overline{X}$ chart (Costa, 1999). The design parameters of the VP $\overline{X}$ control chart considered can assume two values each as a function of the most recent process information. That is, the position of each sample point on the VP $\overline{X}$ chart establishes the size of the next sample, the instant of its sampling and the width coefficient of the control limits.

We denote the values of the sample sizes by n_1 and n_2, the sampling intervals by h_1 and h_2, and the coefficient used in determining the width for the warning and control limits by w_1 and w_2, k_1 and k_2, respectively.

Let LCL and UCL represent, respectively, the lower and upper control limits for a VP $\overline{X}$ chart. The interval (LCL, UCL) is partitioned into three distinct regions: (LCL, LWL); (LWL, UWL); (UWL, UCL); with $LCL < LWL < UWL < UCL$. Here, LWL and UWL represent, respectively, the lower and upper warning limits of the $\overline{X}$ chart. As in the case of FP control charts, a signal is produced when a point falls outside the control limits. In the same way as in the FP control charts, this signal can be false or true. An alarm is false when a point falls outside the control limits but the mean μ is equal to μ_0. In other words, the control chart signals erroneously the occurrence of an assignable cause.

Note that for each sample point $\overline{x}_j$, $j = 1, 2,...$ twopossibilities will be provided for the warning and control limits (that is, $\mu_0 \pm w_i\,\sigma / \sqrt{n_i}$ and $\mu_0 \pm k_i\,\sigma / \sqrt{n_i}$, $i = 1, 2$, respectively)

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because each design parameter can assume two values. Here, w_1 (with $w_1 < k_1$) and w_2 (with $w_2 < k_2$) denote the width coeficients of the warning limits and they are also design parameters.

In a general form, the functioning policy of the VP $\bar{X}$ control charts establishes the action that should be taken when the sample points are obtained. In particular, this policy can establish that a new sample should be taken and which design parameters values should be utilized for the next sample taking into account the information due to the samples until the present moment. In the model considered, the decision of which design parameters values should be utilized will depend only on the last sample value and in which region, of the control chart, the sample point falls. That is, the design parameters of the control chart vary in function of the most recent process information (for details see, De Magalhães et al., 2002).

The size of the first sample that is taken from the process when it is just starting or after a false alarm, is chosen at random, according to probabilities given below. If the sample was chosen to be large (small) it should be sampled after a short (long) time interval. During the in-control period all samples, including the first one, have probability p_0 of being small and $(1 - p_0)$ of being large, where

$$p_0 = P(|Z| < w_1 \ | \ |Z| < k_1) = P(|Z| < w_2 \ | \ |Z| < k_2) \quad and \quad Z \sim N(0,1) \tag{1}$$

The user might prefer to fix the size of first sample (large or small). If the time between occurrence of assignable cause is long (e.g., small λ) the $\bar{X}$ chart properties are independent of the size of the first sample.

Economic-Statistical Design Model for the Adaptive $\bar{X}$ Control Chart

To control the quality of a process through a control chart costs are incurred. In the model considered (De Magalhães et al., 2002), the expected cost per time unit (*ECTU*) is utilized to analyse these costs. The *ECTU* is a function of the costs incurred in different phases of the production cycle and also a function of the design parameters of the control chart. The expected cost per time unit is minimized with respect to the design parameters of the control chart considered.

To develop the economic model, assumptions about the production process are made. These assumptions characterize the class of production processes to be analysed. Although several supositions have been made, different production processes can be yet appropriatelly modelled.

Assumptions: It is assumed that the samples are independent, and that the process starts in a state of statistical control with mean $\mu = \mu_0$ and later on, due to occurrence of an assignable cause, the process mean goes to $\mu = \delta\sigma$. The length of time the process stays in control is an exponential random variable with mean $1/\lambda$. That is, the assignable cause occurs according to a Poisson process, with a intensity of λ occurrences per time unit.

The process is not self-corrective. During the search for an assignable cause and/or during repair the process may continue in operation or not. The parameters μ, σ and δ are assumed to be known and the parameters to be determined are n_1, n_2, h_1, h_2, w_1, w_2, k_1 and k_2.

The Cost Model

Since the process considered is a renewal reward process (see, Ross, 1970), the *ECTU* can be written as the ratio of the expected cost per cycle *(E(C))* to the expected cycle time *(E(T))*, that is: $ECTU = E(C) / E(T)$.

In the computation of *E(C)* and *E(T)*, the expressions for some variables are dependent on the VP policy adopted.

The expressions for *E(C)* and *E(T)* are given by:

$$E(T) = \frac{1}{\lambda} + (1-\delta_1)E(T_{af}) + E(T_{fc}) + E(T_a) + E(T_{esp}) + E(T_R)$$

$$E(C) = \frac{1}{\lambda}C_0 + C_1[E(T_{fc}) + E(T_a) + \delta_1 E(T_{esp}) + \delta_2 E(T_R)] + YE(F) + W + E(C_{am})$$

Here, $1/\lambda$ represents the average time the process stays in control. $E(T_{af})$ represents the average time spent in the investigation of false alarms and δ_1 is an indicator variable, when the process continues in operation during the search of an assignable cause $\delta_1 = 1$, otherwise $\delta_1 = 0$. The expected time searching for false alarms $E(T_{af})$ is equal to the expected search time associated with a false alarm (T_0) times the expected number of false alarms *E(F)*. $E(T_{fc})$ represents the average time since the occurrence of a shift in the process mean until the chart gives a signal. The average time to analyze a sample is represented by $E(T_a)$. The average time to find an assignable cause is represented by $E(T_{esp})$, it is assumed that this time is equal to a specified time T_*. The average time to do a repair is $E(T_r)$ and also it is assumed to be equal to T_{**}. Note that T_* and T_{**} can count or not to *E(C)* because the expected cost of producing non-conformities while the process is operating out of control is dependent on whether the production process stops or not during the search for an assignable cause and/or during repair. These possibilities are represented by the indicator variables δ_1 and δ_2 ($\delta_2 = 2$, if production continues during repair and $\delta_2 = 0$, otherwise). C_0 and C_1 represent, respectively, the costs per hour due to non-conformities produced while the process is in control and out of control.

YE(F) represents the cost due to false alarms, where *Y* is the cost per false alarm and *E(F)* is the expected number of false alarms. To determine *E(F)* it is necessary to compute the expected number of samples taken during an in-control period. The expected cost of finding and eliminating an assignable cause when one existss is given by *W*; this quantity includes any downtime that is appropriate, and is assumed to be policy independent. The expected cost of sampling and inspection is given by $E(C_{am})$.

This is a general economic model (Lorenzen and Vance, 1986), the explicit expressions for each entry in *E(T)* and *E(C)*, for the VP chart considered, were developed by De

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Magalhães et al. (2002). The explicit expressions of $E(T_{fc})$, $E(T_a)$, $E(F)$, $E(C_{am})$ depend on the design parameters of the chart considered. In $ECTU$, the variables λ, δ_1, δ_2, C_0, C_1, T_*, T_{**}, Y, W are input variables.

Economic-Statistical Model

The speed of detection of a shift in the process mean determines the efficacy of the control scheme. That is, the agility of a control chart in detecting a shift is determined by the length of time to produce a signal.

Usually, the process starts in control and some time in the future a shift occurs in the process mean. This suposition was assumed in the model considered. When a process is in control, it is desirable that the mean time since the beginning of the process until a signal be long; this guarantee few false alarms. This mean time is denoted by ATS_0. The expression of the ATS_0 for the VP control chart is given by

$$ATS_0 = \frac{[h_1(1-p_{22})+h_2 p_{12}]p_0 + [h_2(1-p_{11})+h_1 p_{21}](1-p_0)}{1-p_{11}-p_{22}+p_{11}p_{22}-p_{12}p_{21}}$$

where

$$p_{11}(0) = P(-w_1 < Z < w_1)\,,\; p_{12}(0) = P(-k_1 < Z < -w_1) + P(w_1 < Z < k_1)$$
$$p_{21}(0) = P(-w_2 < Z < w_2)\,,\; p_{22}(0) = P(-k_2 < Z < -w_2) + P(w_2 < Z < k_2)$$

and p_0 is given by Eq. (1).

The ATS's expression for $\overline{X}$ control chart with variable parameters was developed by Costa (1999).

If the process is out of control, then the time since the shift until an alarm occurs should be short, because in such case the off-target condition can be detected quickly. This average time is denoted by $AATS$ (adjusted average time to signal). The $AATS$'s expression is given by

$$AATS = E(R) + E(S)$$
$$E(R) = \{h_1 - \tau_{h1}\}P(A = h_1) + \{h_2 - \tau_{h2}\}P(A = h_2)_z$$

$$\begin{aligned} E(S) = \{&[h_1(1-p_{22}(\delta)) + h_2 p_{12}(\delta)][p_{11}(\delta)P(A=h_1) + p_{21}(\delta)P(A=h_2)] \\ &+[h_2(1-p_{11}(\delta)) + h_1 p_{21}(\delta)][p_{12}(\delta)P(A=h_1) + p_{22}(\delta)P(A=h_2)]\} \\ &\times\frac{1}{1-p_{11}(\delta)-p_{22}(\delta)+p_{11}(\delta)p_{22}(\delta)-p_{12}(\delta)p_{21}(\delta)} \end{aligned}$$

$$\tau_{h_1} = \frac{1-e^{-\lambda h_1}(1+\lambda h_1)}{\lambda\left(1-e^{-\lambda h_1}\right)} \qquad \tau_{h_1} = \frac{1-e^{-\lambda h_1}(1+\lambda h_1)}{\lambda\left(1-e^{-\lambda h_1}\right)}$$

$$P(A=h_1)=\frac{h_1 p_0}{h_1 p_0 + h_2(1-p_0)} \qquad P(A=h_2)=\frac{h_2(1-p_0)}{h_1 p_0 + h_2(1-p_0)}$$

A criticism about economic designs is that they do not take care of relevant statistical properties, for example, the optimal design parameters selected by the economic model can allow an excessive number of false alarms (ATS_0) and long average time since the shift until a signal (long $AATS$).

To improve the efficiency of the control charts, mainly in the detection of small to moderate shifts of the target value (0.5 σ to 1.5 σ), was the primary reason in the development of adaptive control charts (see, for example, Costa, 1998; Costa, 1997; Costa, 1994; Prabhu et al., 1994; Prabhu et al., 1993; Runger and Montgomery, 1993; Runger and Pignatiello, 1991; Reynolds et al., 1990; Reynolds et al., 1988). In fact, the adaptive control charts brought significant improvements in the statistical performance of economic designs (Park and Reynolds, 1994; Prabhu et al., 1997; De Magalhães et al., 2002).

If during the optimization of $ECTU$, constraints are imposed on the expected time to signal when the process is in control (ATS_0) and out of control ($AATS$), this ensures that the required statistical properties are satisfied.

The constraints are $ATS_0 \geq l$ and $AATS \leq u$, where l is a lower bound to ATS_0 and u is an upper bound to $AATS$. The design parameters of the economic-statistical model for the VP $\overline{X}$ control chart are obtained solving the optimization problem

$$\min\ ECTU$$
$$subject\ to \begin{cases} ATS_0 \geq l \\ AATS \leq u \end{cases}$$

Sensitivity Analysis – A Case Study

When an economic-statistical model is considered the effect of the choice of the bounds on ATS_0 and / or $AATS$ on the cost and on the design parameters is natural issue that arises and this is presented below.

The example considered consists of a foundry operation process (Lorenzen and Vance, 1986). The carbon-silicate content of the casting is an important quality characteristic because high carbon-silicate content results in casting of low tensile strength. Periodic samples of molten iron are taken to monitor the carbon-silicate content of the casting.

In this example, the process continues in production during the search for the assignable cause ($\delta_1 = 1$), but it is stopped for repair ($\delta_2 = 0$). The average time the process stays in control is 50 hours. The sampling cost of each piece is \$ 4.22. When the process goes to an out of control condition and the assignable cause is identified, the production process is stopped and should be repaired and re-initialized; this takes about 45 minutes. The search of an assignable cause, independently, if it exists or not, take about of 5 minutes.

The costs per hour due to non-conformities produced while the process is in control and out of control are, respectively, \$ 114.24 and \$ 949.20. The cost per false alarm is \$ 977.40 and it is equal to the cost of repairing. Then, considering the notation introduce, the input parameters are: $1/\lambda = 50$ h; $T_{**} = 45/60$ h; $T_0 = T_* = 5/60$ h; $C_0 = \$\ 114.24$/h; $C_1 = \$\ 949.20$/h; $Y = W = \$\ 977.40$: $C_{am} = a + bn$; $(a = 0;\ b = \$\ 4.22)$.

To accomodate limitations of practical order, the optimization of the unit cost function was accomplished considering the following constraints: $n_1 < n_2$; $n_1 \geq 1$; $n_2 \geq 1$; $0.1 \leq h_2 < h_1$; $h_1 \geq 1$; $0.1 \leq w_2 < w_1$; $1 \leq k_2 < k_1$. A nonlinear constrained optimization algorithm available in MATLAB (MATLAB Optimization Toolbox, 1994) was applied to the cost function.

For each shift of the mean ($\delta = 0.5; 0.75$), the *ECTU* was optimized. Since the convexity of the objective function could not be ascertained, different starting vectors were used in the optimization process to find the minimum value of the *ECTU* and the corresponding optimal design parameters. For each specific shift, all searches converged to the same solution, independently of the given starting vectors, providing evidence that, probably, the global minimum was attained.

Sensitivy Analysis of the Cost and the Design Parameters to the Bounds on ATS_0 and *AATS*

For the example considered, *ECTU* was optimized subject to constraints on ATS_0 and *AATS*. The more stringent lower bound chosen for the ATS_0 was 500 hours and the more relax upper bound considered for the *AATS* was 4 hours. These values were chosen considering literature sugestions (Saniga, 1989; Prabhu et al., 1997). According to the literature, control charts which have an average rate of false alarms greater or equal than 500 hours and detect a shift in the mean of the process in an average time smaller or equal than 4 hours are considered good statistical devices for the process control. Still, according to the literature, when the goal is to detect mean shifts in the interval [0.5; 1.0] standard deviation, control charts having $AATS \leq 8$ h are still considered good statistical devices.

The following analyses were conducted:

Variation of the Upper Bound u on *AATS*. For the shift $\delta = 0.5$, the restriction on ATS_0 was kept fixed ($ATS_0 \geq 500$); but the upper bound u on *AATS* was allowed to vary, in order to show the behaviour of the optimal *ECTU* and the optimal design parameters when more restrictive constraints were imposed on *AATS*.

Six upper bounds (u) for the *AATS* were considered: 1.0 h, 1.5 h, 2.0 h, 2.5 h, 3.0 h, and 4.0 h. *ECTU* was minimized subject to $ATS_0 \geq 500$ and $AATS \leq u$, u taking the values considered above. Then for each restriction considered to the *AATS*, the optimal *ECTU* and design parameters were obtained. These results allowed to build the plots shown in Figure 1. The same kind of analysis was repeated for $\delta = 0.75$. Qualitatively, the same behavior described below for $\delta = 0.5$ was observed.

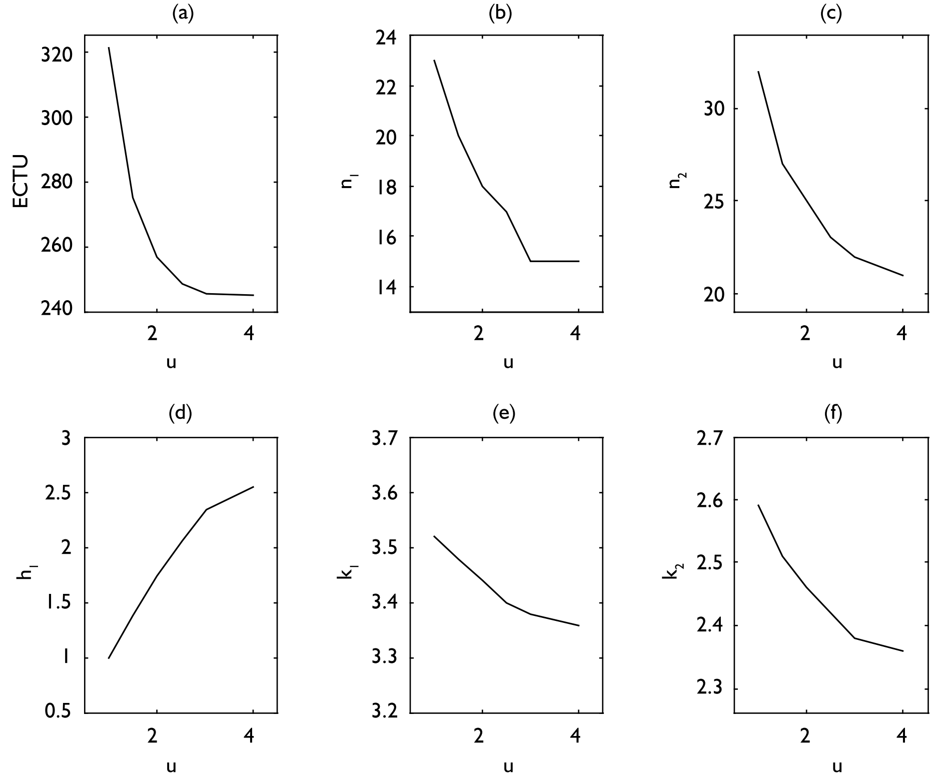

Figure 1 – Effect on the cost and the design parameters due to the bounds on *AATS*.

Variation of the Lower Bound *l* on *ATS_0*. For the shift $\delta = 0.5$, the restriction on *AATS* was kept fixed (*AATS* ≤ 4); but the lower bound on ATS_0 was allowed to vary. Four lower bounds (l) on ATS_0 were considered: 200, 300, 400, and 500 hours. For each lower bound for the ATS_0, *ECTU* was minimized subject to $ATS0 \geq l$ and $AATS \leq 4$. Then, the optimal *ECTU* and design parameters were obtained. The results are shown in Figure 2.

Sensitivity of the Solution to the Upper Bound *u* on *AATS*

Figure 1a shows that tighter constraints on *AATS* increase the *ECTU*, or in another way, as u, the upper bound on *AATS*, increases, the cost (*ECTU*) decreases, becoming insensitive to restrictions on *AATS* from $u = 3$. Note also that the cost decrease is quite rapid around smaller values of u.

Figures 1b and 1c show, respectively, that as u increases the values of the sample sizes n_1 and n_2 decrease. However, from $u = 3$, n_1 became insensitive to restrictions on *AATS*. From Figure 1d it can be noted that as u decreases, h_1 decreases, that is, with tighter restrictions on *AATS* samples are taken more often. From $u = 3$, h_1 became less sensitive to restrictions on

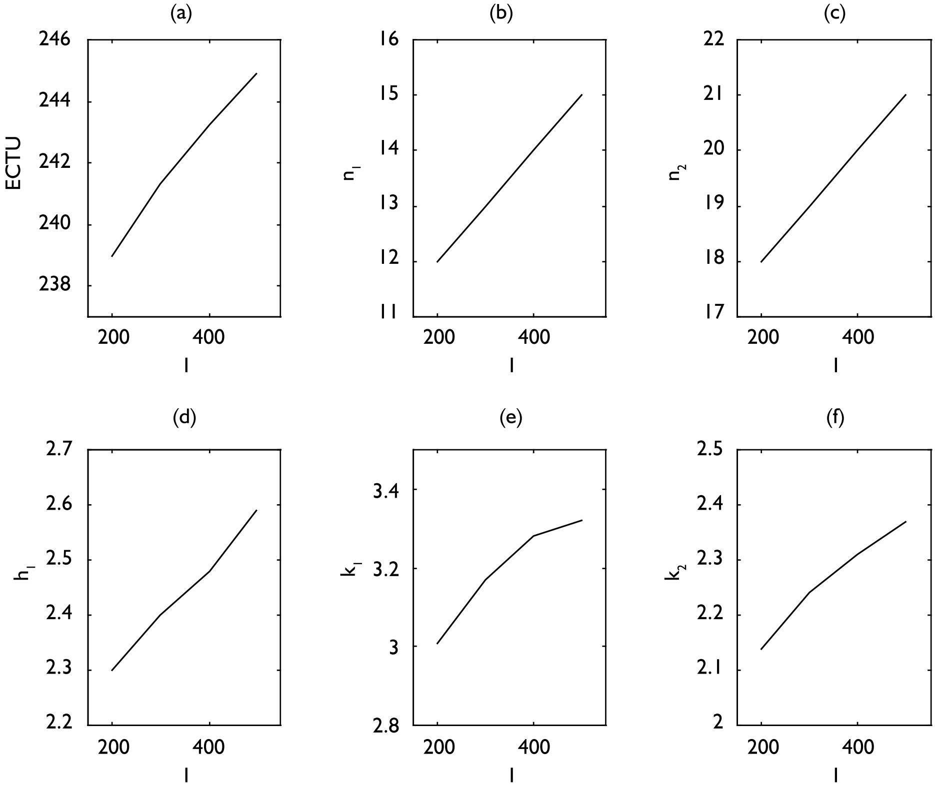

Figure 2 – Effect on the cost and the design parameters due to the bounds on ATS_0.

AATS. The sampling interval h_2 is insensitive to constraints on *AATS*, it assumes always the value 0.1. Figures 1e and 1f show that as u increases, k_1 and k_2 decrease. In fact, when u varies from 1 hour to 4 hours, k_1 decreases from 3.52 to 3.36, and k_2 decreases from 2.58 to 2.36.

As said before, the same kind of analysis was repeated for $\delta = 0.75$ and, qualitatively, the same behavior described for $\delta = 0.5$ was observed. However, it should be mentioned that for each shift (for the shifts considered, $\delta = 0.5$ e $\delta = 0.75$), there is a value, say u^*, for the bound of *AATS* such that for any value u of the bound above u^*, the *ECTU* does not change. This is due to the fact that for bounds above u^*, the optimal values of *ECTU* have *AATS* strictly less than the bound. The value of u^* decreases when δ increases.

Sensitivity of the Solution to the Lower Bound l on ATS_0

Figures 2a and 2d show, respectively, that restrictions on ATS_0 produce an approximately linear and increasing effect on the cost and the sampling interval h_1. Figures 2b and 2c show that restrictions on ATS_0 produce a linear increasing effect on the sample sizes n_1 and n_2.

The sampling interval h_2 is insensitive to restrictions on ATS_0. As before, it assumes always the value 0.1. Figures 2e and 2f show that wider restrictions on ATS_0 make k_1 and k_2 values decrease.

Conclusions

The main goal of an economic-statistical model for control charts is to improve the statistical performance of economic models. Considering the economic-statistical model for VP $\bar{X}$ control charts developed by De Magalhães et al. (2002), in which constraints are imposed on the average times until a signal (ATS_0 and $AATS$), an analysis of the effect of the choice of these bounds on the optimal design parameters and cost was performed. To have an idea of how the design parameters and the expected cost per time unit vary is an important decision factor. In fact, since the bounds on the $AATS$ and on ATS_0 are to a certain extent arbitrary, so if they lead to a large expected cost, the user may try to vary one or both of these bounds to diminish the cost.

To make the paper more relevant to practitioners the following observations are worthy of notice. Once we are working with an economic-statistical model for a VP $\bar{X}$ chart, finding the design parameters is not trivial; however, the results provide some guidelines as how to set control limits, sampling intervals and sample sizes according to the bounds on the $AATS$ and ATS_0 that one is willing to have in a process.

For the process considered, when the process is out of control ($\delta = 0.5$) and if it is desirable to detect this condition in less than 2 hours, then, from the analysis provided, the ranges of the design parameters should be: $18 \leq n_1 \leq 24$, $25 \leq n_2 \leq 35$, $1 < h_1 < 2$, $3.42 < k_1 < 3.52$, $2.46 < k_2 < 2.58$, considering an average rate of false alarms greater than or equal to 500 hours ($ATS_0 \geq 500$ hours). For these ranges of design parameters the $ECTU$ varies between 258 dollars to 320 dollars.

Generalizations based on the results will probably provide more insight to practitioners. For example, the results show that the small sample size (n_1) never falls below 14 (when the bounds on $AATS$ are varying) and never falls below 12 (when the bounds on ATS_0 are varying) for the case studied in this article.

As said before, the primary reason to develop adaptive control charts is to improve the efficiency of the traditional (Shewhart) control charts in the detection of small to moderate shifts of the target value (0.5 σ to 1.5 σ). Therefore, we made a sensitivity analysis for $\delta = 0.5$ to check the performance of the VP $\bar{X}$ chart in the presence of a small shift. The same kind of analysis was repeated for $\delta = 0.75$ and, qualitatively, the results are similar to the ones obtained for $\delta = 0.5$. From this, we surmise that similar qualitative behavior holds for other values of δ.

In this way, even though the article has worked with a specific example, the study provides a useful insight to quality control designers in making the trade-off decision between the expected cost and desired levels of statistical properties.

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Biography

Maysa S. de Magalhães is an Assistant Professor in the Escola Nacional de Ciências Estatísticas at Instituto Brasileiro de Geografia e Estatística, Rio de Janeiro, Brazil. She received a PhD degree from Pontifícia Universidade Católica do Rio de Janeiro. Her current interests are in statistical quality control and times series. She has published papers in *the International Journal of Production Economics, European Journal of Operational Research, International Journal of Production Research, and (in Portuguese) in Gestão & Produção*. She is also a member of the *Sociedade Brasileira de Pesquisa Operacional (SBPO)* and a reviewer for some journals.

Antonio F. B. Costa is an Assistant Professor in the Department of Production at UNESP- São Paulo State University, Brazil. He received a PhD degree from State University of Campinas, Brazil. He was a postdoctoral fellow in the Center for Quality and Productivity Improvement at University of Wisconsin, Madison, USA. His current interest is in statistical quality control. He has published papers in the Journal of Quality Technology, Communications in Statistics, IIE Transactions, International Journal of Production Research, International Journal of Production Economics, Quality Reliability Engineering International, Quality Technology and Quantitative Management, Journal of Applied Statistics, European Journal of Operation Research, Quality Engineering, and (in Portuguese) in *Gestão & Produção* and *Pesquisa Operacional*. He received a Best Paper Award from IIE Transactions. He is a Certified Quality Engineer by the American Society for Quality and a member of the *Associação Brasileira de Engenharia de Produção* (ABEPRO) and of the *Sociedade Brasileira de Pesquisa Operacional* (SBPO).