Brazilian Journal of Operations & Production Management

ISSN 2237-8960 (Online)

ABEPRO ASSOCIAÇÃO BRASILEIRA DE ENGENHARIA DE PRODUÇÃO

RESEARCH PAPER

Control charts for monitoring process with time trend: using monitoring random source, profile monitoring and modified location chart

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How to cite: Oprime, P. C.; Vicentin, D. C.; Sordan, J. E. (2025), "Control Charts for Monitoring Process with Time Trend: Using Monitoring Random Source, Profile Monitoring and Modified Location Chart", *Brazilian Journal of Operations and Production Management*, Vol. 22, No. 3, e20252417. https://doi.org/10.14488/BJOPM.2417.2025

ABSTRACT

Purpose: Statistical process monitoring has been a relevant practice incorporated into quality management systems. In a controlled process, the variabilities of statistical parameter estimates are expected to fluctuate within a pattern over time. Whenever this pattern is not identified, the root causes must be identified, and the necessary actions should be taken. However, there are situations where special causes are present but not practically significant. Recent studies on profile monitoring have tailored solutions for efficiently detecting trends and seasonality in high-capability processes. This paper proposes using profile curves and statistical modelling based on the location control chart approach.

Design/methodology/approach: Our research was carried out using decision-prescriptive models and the Design Science Research approach to identify solutions for real and complex problems. Additionally, the modeling and simulation method was utilized to assist in developing, analyzing, and testing the model, which was classified as an artifact.

Findings: The modeling results clearly demonstrate that utilizing location control charts and modified graphs is essential in defining equipment adjustments. This approach guarantees the minimum acceptable capacity and maximizes the use of productive resources.

Originality: This study provides novel opportunities for developing and implementing control charts in systems that do not conform to the principles of randomness but rather display intricate temporal patterns. This is particularly relevant in the context of Quality 4.0, where real-time data collection is pervasive.

Keywords: Statistical Process Monitoring; Location Control Charts; Profile Monitoring; Quality 4.0.

1 INTRODUCTION

Statistical Process Monitoring (SPM) has been a practice incorporated into quality management systems for many years. First developed by Shewhart in the 1920s, statistical process control brought innovation to the way processes are managed. The statistical identification of two different types of variability causes (random and special) at that time was groundbreaking and revolutionary, earning it a place as one of the key techniques in Total Quality Management (TQM). Its statistical principles are based on extracting random samples in subgroups at intervals. The assumption is that only random causes are present within the subgroups, and special causes are present between subgroups (Woodall and Montgomery, 2014). For a process to be in control, only random causes should be present. Thus, a quality characteristic is monitored over time by continuously assessing a location parameter (central tendency) and a scale parameter – variation (Srikaeo and Hourigan, 2002; Mohammadian and Amiri, 2012; Mukherjee, 2015; Goedhart and Woodall, 2022).

Financial support: BUP Research Centre (BRC) under Bangladesh.
Conflict of interest: The authors have no conflict of interest to declare.
Corresponding author: damaris_sv@hotmail.com
Received: 09 October 2025.
Accepted: 21 July 2025.
Editor: Osvaldo Luiz Gonsalves Quelhas.

Despite the numerous published works, practical issues have more recently gained attention. There are situations where special causes are present but not practically significant. This is easily observed in processes with high capability indices, Cp and Cpk. Literature has solutions for this type of problem, known as acceptance charts or modified charts. Other recent solutions have been presented, incorporating Cp and Cpk indices into traditional Shewhart charts and introducing the concept of practical significance in statistical modeling in CUSUM and Weighted Moving Average charts. Woodall (1985) asserts that small changes in the process perceived over time may have little or no practical importance. Similarly, Mohammadian and Amari (2013), and Oprime and Mendes (2017), suggest that in highly capable processes, where natural variation is much smaller than specification limits, control limits should be relaxed, allowing the mean to vary within a certain range of values.

The pioneer in this area was Freund, who introduced acceptance charts in 1957, as the author himself indicated, suitable for high-capability processes (Mhatre *et al.* 1981; Holmes and Mergen, 2000; Mohammadian and Amiri, 2013; Woodall and Faltin, 2019). These authors contributed to the development and practical use of modified and acceptance control charts. However, a practical issue mentioned in the literature is the non-random behaviour of process measurements over time, the time trend, which could be seasonality, trend, or any other non-random profile. Understanding process profiles, when they exist, including the concept of practical significance, is an aspect that deserves attention from theorists and users of the technique. For this reason, new methods and types of control charts must be developed. According to Shper and Adler (2017), there is a potential existence of unknown (implicit) patterns in any given process. These patterns can either influence or not influence the process outcomes, but the challenge lies in the uncertainty of their presence.

To overcome these limitations, more recent approaches, such as Profile Monitoring and specific techniques for location and variation control, offer more tailored solutions for efficiently detecting trends and seasonality in high-capability processes. Including these advanced methodologies allows for a more robust and sensitive understanding of changes in process behavior promoting a proactive response to non-random alterations. Kang and Albin (2000) pioneeringly argue that there are situations in which the quality of the process or product can best be characterized by a functional relationship between the response variable, corresponding to the quality characteristic of interest, and one or more explanatory variables.

Linear temporal trends arise in the presence of uncontrolled factors and invalidate the assumption of independence among different values of the response variable. The effects of linear trends have already been studied in experimental design by Draper and Stoneman (1968), Cheng and Jacroux (1988), and Hilow (2013), for example. This way of considering the effects of linear trends in DoE changes the traditional proposal of randomization. We present this subject here to draw an analogy with control charts. Therefore, we can also mention here that a systematic order of sampling the process on a control chart should be considered in practice.

Our motivation for addressing this issue stems from the challenges faced by engineers and production supervisors in effectively analyzing critical process characteristics using traditional Shewhart charts and capability calculations. The aim of this research is to utilize decision-prescriptive models to identify solutions to novel problems or compare the efficacy of strategies to address a given issue. In addition, the study proposes the use of profile curves and statistical modeling based on the location control chart approach and residues to monitor processes with time trends. To achieve this goal, the paper will employ Design Science Research (DSR) methodology to solve practical problems through mathematical modeling. This approach emphasizes the importance of creating practical and effective solutions, as well as contributing to the advancement of knowledge in statistical process monitoring.

This paper has the following structure, in addition to the introduction: Section 2 presents what we understand as processes with linear trends, adjustments of a polynomial model, and analysis and monitoring of their residuals. Section 3 presents the research method. Section 4 introduces the results of the monitoring model based on the profile and location control chart, exploring theoretical and practical implications of the developed study. Finally, Section 5 provides the conclusions regarding the proposed approach.

2 EFFECTS OF LINEAR TRENDS AND RESIDUAL MONITORING

2.1 Contextualization of the problem of Linear Trend Effects in manufacturing processes

Shper and Hard (2017) bring an interesting discussion about the importance of temporal order as an aspect to be considered in Phase I implementation of control charts for process monitoring. According to these authors, this aspect is often overlooked in the literature because there are situations where processes exhibit intrinsic trends or seasonality. From a practical standpoint, such temporal trends may be acceptable.

In Phase I analysis, data is used retrospectively to assess process stability and establish limits that will later be used in Phase II for prospective monitoring. In Shewhart charts, any non-random behavior indicates the presence of special causes. Shper and Hard (2017) argue that patterns exist in all real-world processes, but sometimes their influence may be small enough to ignore. However, in practice, there are cases where non-random trends are relevant and easily identified or known beforehand.

We often encounter situations where engineers attempt to apply the traditional control chart to processes with linear trends, i.e., without randomness. When this happens, capability studies are compromised. The method of estimating the process standard deviation must be considered from the perspective of the analyzed phenomenon. There are many practical constraints and considerations when applying control charts in the presence of temporal trends or non-random events. The practical sense and the impact of linear trends are crucial for practitioners. Woodall (1985) mentioned that small changes in the process, observed over time, may have little or no practical significance.

This view is supported by Mohammadian and Amari (2012) suggest that in highly capable processes, where natural variation is much smaller than specification limits, the control limits should be relaxed, allowing the mean to vary within a certain range of values. Similar arguments are found in Kuiper and Goedhart (2023) for CUSUM and EWMA charts. Practical considerations must be taken into account when using control charts. In the words of Woodall and Faltin (2019), a slight deviation should not always be considered an out-of-control situation. From a practical standpoint, even the presence of linear trend effects, if they do not produce significant impacts, can be deemed acceptable as long as they are monitored.

Building upon this perspective, recent research highlights the potential of auxiliary information-based (AIB) charts (Aslam *et al.*, 2022), to enhance detection power, especially in scenarios involving small shifts. These developments reflect a growing consensus that monitoring strategies must be adapted to reflect the inherent characteristics of industrial processes, particularly those governed by deterministic structures such as tool wear or gradual equipment drift.

In response to this need, various authors have contributed to expanding the theoretical and practical frameworks for monitoring such systems. For instance, profile monitoring has emerged as a promising alternative. Pioneering studies by Kang and Albin (2000) and Colosimo and Pacella (2010) highlight the effectiveness of modeling quality characteristics as functions of one or more explanatory variables. This approach has been further validated by Noorossana *et al.* (2011) and Eyvazian *et al.* (2011), who demonstrate the usefulness of regression and residual analysis in capturing nonlinear or structured variation in process data.

Moreover, the linear temporal trends arise in the presence of uncontrolled factors and invalidate the assumption of independence among different values of the response variable (while the independence of control variables is guaranteed by the orthogonality of design matrices). The effects of linear trends have been studied in experimental design (Draper and Stoneman, 1968; Cheng and Jacroux, 1988; Hilow, 2013; Pureza *et al.*, 2018). This way of considering linear trend effects in Design of Experiments (DoE) alters the traditional randomization approach.

In summary, statistical monitoring advocates for hybrid models that integrate traditional process monitoring techniques with regression, profile modeling and practical significance thresholds. These approaches not only increase robustness in detecting meaningful variations but also align monitoring practices with the operational realities of high-precision and non-stationary processes.

The following subsections provide a more detailed theoretical foundation for the models proposed in this study to integrate temporal patterns and profile behavior into statistical process monitoring.

2.2 Approach for Monitoring Processes with Time Trends Using Residual Control Charts

Residual analysis plays a fundamental role in evaluating the adequacy and accuracy of fitted regression models. It enables the identification of discrepancies between model predictions and observed data, thus supporting model refinement (Altun, 2020; Montgomery, 2019; Box *et al.*, 1978). When the underlying assumptions of the model are satisfied, residuals are expected to exhibit random variation with a mean of zero, or approximately zero, in empirical applications.

In this context, control charts provide a complementary analytical approach by allowing residual behavior to be monitored over time. If the residuals remain randomly distributed and within control limits, the model is deemed appropriate for practical use. Consequently, the model can be employed to determine whether a process remains in control or has deviated from its expected behavior, based on the accuracy of the estimated profile (Pereira *et al.*, 2020).

Control charts that incorporate linear trends are often appropriate for practical applications for two key reasons: (a) they tend to exhibit high process capability, which can be easily assessed by comparing engineering tolerances with process variability. This capability can be numerically verified by estimating the residual standard deviation, which reflects variability arising from unknown or uncontrolled factors. Such estimation is straightforward when using linear regression techniques; and (b) the dominant sources of process variability are known and manageable.

Processes affected by tool wear exemplify the type of scenario addressed in this study. In these cases, while short-term variability tends to remain within permissible limits, tool degradation over time leads to a gradual upward shift in the process mean, resulting in systematically larger measurements. Typically, capability indices such as Cp and Cpk remain high throughout the process—often exceeding the commonly accepted threshold of 1.33. When the wear rate is known, control limits can be adjusted to follow the wear trend line (Montgomery, 2019), such that fluctuations within these adjusted boundaries indicate a process under control.

Woodall (1985) emphasized that small process changes observed over time may lack practical significance. In a similar vein, Mohammadian and Amari (2012) argue that in highly capable processes, where natural variability is substantially smaller than the specification limits, control limits can be relaxed, allowing the process mean to fluctuate within a specified range without compromising quality.

Regression control charts can also be adapted to address tool wear problems. In such cases, both the wear rate and the residuals can be monitored jointly, allowing for control of the wear rate and the variation around each point due to random causes. Residual charts should always be examined to validate a regression model. One alternative for analyzing the model is residual scaling, calculated as: $d_i = \frac{e_i}{\hat{\sigma}}$, where d_i is the value of the scaled (or standardized) residual for the residual e at point i. These residuals have a mean of zero and a variance of approximately one, $\hat{\sigma}$ is the estimated standard deviation of the residuals. Most of the standardized residuals should fall between -3 and 3.

As a tangible illustration of this real-world problem, we present a comprehensive study of a machining process, where the manufactured product at this machining center is designed to meet specifications of 65.6 + 0.2 millimeters. This case illustrates the significant impact of trends, as depicted in Figure 1. From the perspective of the classical approach, this is an unstable process, therefore out of control, as it does not exhibit random behavior. However, from a practical standpoint, it is a highly capable and in-control process. Despite having a systematic trend over time, due to the presence of a known source of variation, this process can be considered stable if this trend reproduces over time. Another important point, which aligns with the assertions of Shper and Adler (2018), is that statistical monitoring and capability studies cannot be determined by the classical approach, as the sample is not random.

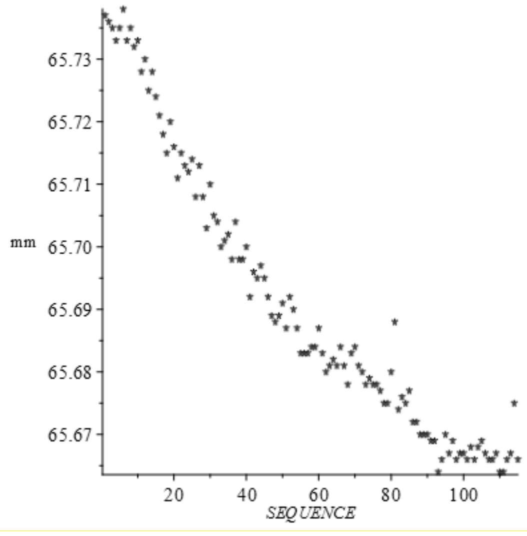

Figure 1 - Samples of 115 parts extracted sequentially

The data plotted on the Y-axis, as indicated in Figure 1, represents the measurements of the piece in millimeters obtained as a function of the production order indicated on the X-axis. The first piece produced (x=1) had a Y-value of 65.737, and the 115th piece had a Y-value of 65.666. Processes with the characteristics shown in Figure 1 should have a different treatment from the approach of Shewhart charts. Monitoring for these cases can consider three measures: i) monitoring of residuals due to unidentified random causes; ii) monitoring of the adjusted polynomial profile; and finally; iii) monitoring of the angular coefficient derived from the rate of variation of Y with respect to production sequence X ($\lim_{x \to x_0} \frac{f(x)-f(x_0)}{x-x_0}$).

The first step is to develop a mathematical model that can be used to infer the response from a given combination of factor values. If X_i are known quantities for each experimental run, models with p parameters are generally represented by a multiple regression model. For the case of Figure 1, we can consider that the independent variable is associated with time t, due to the sequential order of process sampling. Fitted to data similar to that in Figure 1, a statistical model would be a polynomial of the form:

$$\hat{\mu}_{(Y|X_0)} = \beta_0 + \beta_1 X_0 + B_{11} X_0^2 \quad (1)$$

Considering X as an integer ranging $1 \leq X \leq m$ (where m represents the total number of measured pieces), the production sequence would act as an auxiliary predictor variable of the inspected quality characteristic. We should consider that the adjustment of the mathematical model will be suitable to establish the relationship between the dependent variable Y (the quality characteristic) and the independent variable X (which is the manufacturing order of the pieces). In this case, the error (or residual) represents the variability due to unknown causes, with a Gaussian (normal) probability distribution, with zero mean and standard deviation σ_e. As already known, the random error is obtained by the difference between the observed value of Y for a given value of X minus the predicted value, $\hat{\mu}_{(Y|X_0)}$. Thus, the estimated standard deviation of the variability due to random causes is easily calculated by:

$$\hat{\sigma}_e^2 = \frac{\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2}{m-p} \quad (2)$$

Where $\hat{Y}_i = \hat{\mu}_{(Y|X_0)}$ is the expected mean value predicted by the model, p is the number of estimated parameters of the regression model. In the example, there are three estimated parameters. We can then apply the classical approach of Shewhart charts for the upper and lower control limits of the residuals using variance control chart, obtained respectively by: $LCL = \frac{\varphi^2_{m-1,\alpha/2}}{m-1}\hat{\sigma}_e^2$ and $UCL = \frac{\varphi^2_{m-1,1-\alpha/2}}{m-1}\hat{\sigma}_e^2$. Actually, $\varphi^2_{m-1,\alpha/2}$ and $\varphi^2_{m-1,1-\alpha/2}$ denote the $\alpha/2$ quantile and $1-\alpha/2$ quantile, respectively, of the distribution of a chi-square variable with m − 1 degrees of freedom. LCL is Lower Control Limits, UCL is Upper Control Limits, α is significance level. Since $\hat{\sigma}_e^2$ is an estimated parameter, LCL and UCL are also estimated control limits ($\widehat{LCL}$;$\widehat{UCL}$).

Generally, for a Type I error of α =0.0027 (0.27%). The residuals are iid normal random variables. In Phase II, any change in one of the parameters of the model from equation 1 adjusted to the data in Phase I would be detected in the residual plot. Considering that the random error $e_i = (Y_i - \hat{Y}_i)$ has a normal distribution with zero mean and standard deviation $\sigma_e, N \sim (0, \sigma_e)$ and they are independent, where i = 1, 2, ..., m. Assuming that the mean is known and zero and σ_e is estimated by Equation 2, the process is under control when the residual is within the control limits.

However, in practice, we estimate the mean error using Equation 3 by:

$$\bar{e} = \frac{1}{m}\sum_{i=1}^{m} e_i, \quad (3)$$

Also, the mean error is estimated using the variance of the residuals with Equation 3. However, we scaled (standardized) residual by $d_i = \frac{e_i}{\hat{\sigma}}$ (see section 2.2), so that the control limits are calculated by: Center Line, $CL = \frac{\bar{e}}{\hat{\sigma}_e}$, $UCL = CL + K\hat{\sigma}_r$ and $LCL = CL - K\hat{\sigma}_r$, where $\hat{\sigma}_r$ is the estimated standard deviation of the residual, which is approximately 1, and K=3 (standard deviation number). For the case, $CL = 0.407, LCL = -2.58, UCL = 3.39$, $\hat{\sigma}_r = 0.9945$. The residual control chart presented in Figure 2 utilizes standardized residuals, as recommended in the literature, to reduce the influence of differing variable scales and ensure comparability (Montgomery, 2019). As shown in the figure, one observation lies outside the control limits, suggesting the presence of a special cause that warrants further investigation.

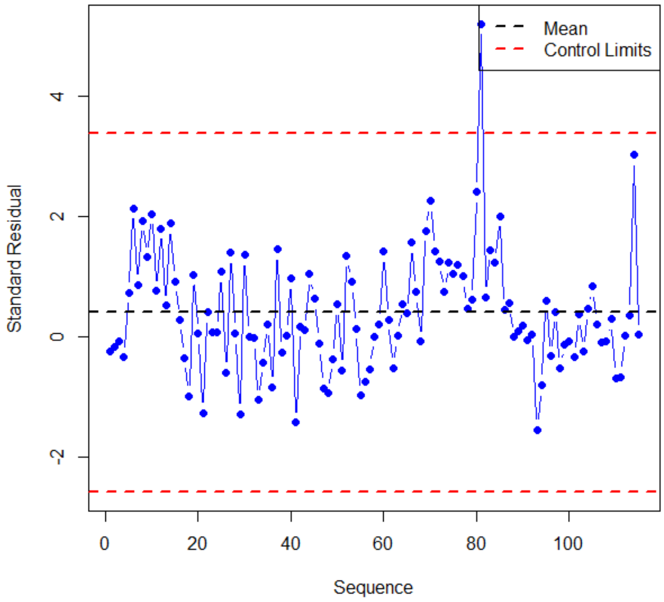

Figure 2 - Scores Residual Control Chart from figure 1.

2.3 Profile monitoring and modified location control chart

The profile model in Figure 1 illustrates how the peculiarities of each process must be considered when designing a monitoring method. In the previous section, we used residuals to analyze and monitor sources of variation. We also proposed equations to assess the performance of residual control charts, adapting them from existing literature. Monitoring residuals has been presented in the literature for profile cases and, therefore, validated. An analysis and discussion on this matter were presented by Noorossana*et al.* (2011) and Eyvazian *et al.* (2011). Colosimo and Pacella (2010) expanded the range of profile monitoring options. These authors demonstrate that most approaches to profile monitoring proposed in the literature share a typical structure, consisting of: i) identifying a parametric model of functional data; ii) estimating the model parameters; and iii) designing a multivariate control chart for the estimated parameters and a univariate control chart for residual variance. The proposed approaches can then be classified according to the type of application faced (i.e., calibration study, process signal, or monitoring of geometric specifications) or the modeling approach considered (mainly linear regression or approaches for reducing multivariate data, such as principal/independent component analysis).

Given that $E\left(\hat{\mu}_{(y|x)}\right) = \mu_{(y|x)}$ and the estimator for σ_e is $\hat{\sigma}_e = \sqrt{\frac{\sum_{i=1}^{n}(Y_i-\hat{Y}_i)^2}{n-p}}$ we can estimate for each point of X the estimated control limits for Y as follows:

$$\widehat{LSC}_{(y|x_0)} = \hat{\mu}_{(y|X_0)} + K\hat{\sigma}_e \tag{4}$$

$$\widehat{LIC}_{(y|x_0)} = \hat{\mu}_{(y|X_0)} - K\hat{\sigma}_e \tag{5}$$

Since there are m points in X for which control limits are estimated for Y, theoretically, there is a heightened probability of a point falling randomly outside the control limits, consequently increasing the risk of Type I error. This suggests that the percentile of the standardized normal distribution used to calculate K should be adjusted. When multiple hypotheses are tested, the likelihood of observing a rare event increases, thereby elevating the probability of erroneously rejecting a null hypothesis (i.e., committing a Type I error). To address this concern, the Bonferroni correction rule for dependent events is utilized to establish an actual false alarm rate that does not surpass a predefined threshold value.

In obtaining Bonferroni intervals, it is not necessary for all separate confidence coefficients $[100(1-\alpha_i)\%, i = 1,2,..,m\,]$ to be equal, but rather that $\alpha = \sum_{i=1}^{m}\alpha_i$. Thus, $\alpha' = \frac{\alpha}{m}$, recalling that m is the number of points in the regression, α is the Type I error of the control chart, and α' is the corrected value of the Type I error for each Y point estimated by the regression, which corresponds to the new K' of the standard normal distribution percentile, as mentioned by Chakraborti (2000), Chakraborti (2000), and Jardim *et al.* (2020). And the ARL can be calculated by:

$$\text{ARL} = \int_{-\infty}^{\infty}\int_{0}^{\infty}\frac{1}{1-\left(\emptyset_i\left(\frac{W}{\sqrt{m}}+K'U-\delta\sqrt{n}\right)-\emptyset_i\left(-\frac{W}{\sqrt{m}}-K'U-\delta\sqrt{n}\right)\right)^m}f(u)\varphi(w)dudw \quad (6)$$

Figure 3 illustrates how the profile chart would appear when adopting the Location Control Chart method for data from figure 1. Figure 3 illustrates the central line along with the upper and lower control limits of the curve that represents the relationship between the production sequence and the diameter measurement of the part.

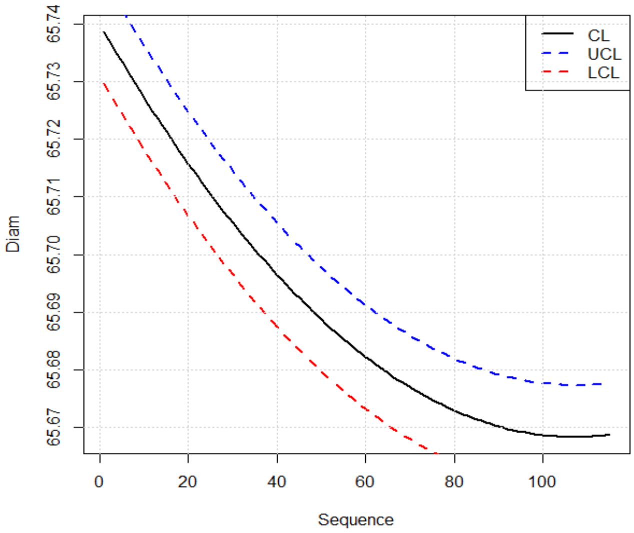

Figure 3 - Profile Monitoring Control Chart Data of Figure 1.

2.4 Control Charts and Performance

The implementation of Statistical Process Monitoring (SPM) traditionally involves two phases (Jones-Farmer *et al.*, 2017). Phase I focuses on defining the quality characteristic and on collecting and analyzing data to estimate statistical parameters, such as the process mean and the standard deviation of the mean, which will subsequently be monitored using control charts. Ideally, in this phase, both the target parameter and its standard deviation are known. However, statistical parameters are not always available a priori. In traditional Shewhart control charts, two parameters are monitored statistically: the mean and the standard deviation.

Based on the availability of these parameters, four scenarios or cases can be considered: a) Case 1: both the mean and standard deviation are known – Known/Known (KK); b) Case 2: the mean is unknown and the standard deviation is known – Unknown/Known (UK); c) Case 3: the mean is known and the standard deviation is unknown – Known/Unknown (KU); d) Case 4: both the mean and the standard deviation are unknown – Unknown/Unknown (UU). When one or both parameters are unknown, Phase I is used to estimate them. Consequently, the upper and lower control limits (UCL and LCL, respectively) are calculated based on these estimates.

The expected control charts' performance reflects α e β errors; α error is the false-positive rate (when the process is in control, but the value found for the monitored parameter is out of limits). β error is the false-negative rate (when the process is out of control, but the value found for the monitored parameter falls within limits). Thus, the smaller the accepted α error, the greater the β error. For h = 3, known parameters and process under control, one point in 370 on average falls outside the calculated limits.

The most common measure for calculating the control charts' performance is the Average Run Length (ARL), where the number of process samples must be evaluated until a point outside the control limits occurs. Due to this characteristic, the sequence size (Run Length – RL) is a random variable that follows a geometric distribution with mean $1/p$, where p is the probability of a successful event (Acosta-Mejia, 1999). For process situations under control, this value is expected to be as high as possible. In Equation 7, p = α, and usually α = 0.0027 in control charts. Thus, ARL_0= 370 for a process with known μ and σ.

$$ARL_0 = 1/\alpha \quad (7)$$

In out-of-control process situations, such value is expected to be as low as possible, quickly identifying abnormal variations in the process. In this case, the power of the test is used to calculate the ARL shown in Equation 8 (Sobue *et al.*, 2020; Jones-Farmer *et al.*, 2014; Haridy *et al.*, 2011; Jensen *et al.*, 2006; Woodall, 1985). It is important to emphasize that in Phase I (retrospective phase), the main interest is to understand better the process and evaluate its stability. In contrast, Phase II (prospective phase) involves controlling a process by historical data analysis eliminating any attributable variation causes (Chakraborti *et al.*, 2009).

$$ARL_{OOC} = 1/(\text{test power}) = 1/\ (1- \beta) \quad (8)$$

3 METHOD

In this article, the problem addressed is not fundamentally theoretical but rather empirical, requiring the adaptation of the traditional Shewhart control chart approach. The main characteristic of the problem here is that the process studied does not exhibit stationary behavior, a foundation of the classical approach, which aims to keep the process stationary over time.

The process intrinsically displays non-stationary behavior, necessitating an adaptation of the known monitoring method. Two approaches were applied. The first is a prescriptive approach, where we developed analytical models considering the effects of linear trends, natural to the process, and not viewed as a special cause to be corrected. This modeling strategy aligns with approaches in the literature that jointly optimize production and quality control systems (Liao *et al.*, 2017; Zhang *et al.*, 2022). These prescriptive frameworks have been increasingly applied to scenarios such as predictive maintenance, process monitoring, and hybrid classification problems in operations research (Yan *et al.*, 2024). The second is the implementation of the Design Science Research (DSR) solution, which is particularly suited for developing and evaluating artifacts that solve real-world problems (Hevner *et al.*, 2019). DSR enables a structured process of problem-solving by integrating theory and practice, emphasizing relevance and rigor in both the artifact design and its contextual application (Carstensen & Bernhard, 2018). In this study, DSR is used to guide the development of new control chart models adapted to processes with time trends, enabling iterative refinement and evaluation based on empirical evidence.

In summary, our research was conducted using prescriptive decision models and the Design Science Research approach to identify solutions for real and complex problems. Additionally, modeling and simulation methods were used to support the development, analysis, and testing of the model, which was classified as an artifact.

Thus, we generated innovative solutions for a lesser-studied problem: processes with linear trend effects, which are, therefore, non-stationary. The created and tested artifacts, namely statistical models and methods were designed to solve the specific problem. The artifacts were evaluated in terms of their effectiveness and usefulness in real contexts.

From a methodological standpoint, we followed these steps: i) extracted a sample of 125 pieces in sequence and plotted this data in time order; ii) fit a second-degree polynomial to the data to interpolate linear trend effects; iii) calculated the residuals of the fitted model to establish stationarity; iv) adjusted the control chart limits for the residual plot, modeled the residuals chart, and evaluated its performance in terms of ARL; v) developed the temporal profile model and evaluated its performance; vi) derived the rate of change function for the process by differentiating the profile function; vii) proposed as a final solution the creation of the Modified Control Chart applied to the Profile as a solution to the studied problem.

4 EMPIRICAL APPLICATION: A REAL CASE ANALYSIS

4.1 Modeling of control charts for residuals

To adjust an empirical model to the data in the graph of Figure 1, the Maple program was used with the $PolynomialFit(2, X, Y, v)$ function, which resulted in the following Equation 9.

$$\mathrm{y} = 65.74 - 0.001339v + 6.252 * 10^{-6}v^2 \quad (9)$$

Table 1 presents the outcomes of significance tests conducted on the model parameters, affirming their statistical significance. The coefficient of determination (R-squared) for the model is 0.9815, signifying a robust explanatory power. Additionally, the estimated residual standard deviation stands at 0.00301, underscoring the model's accuracy in predicting observed data points. Furthermore, the standard errors of the model parameters were determined $[0.000856963;\ 0.0000341;\ 2.8481\ 10^{-7}]$.

Table 1 - Coefficients of model

	Estimate	Std. Error	t-value	P(>|t|)
Parameter 1	65.7402	0.0008569	76713.0	0.0000
Parameter 2	-0.0013	0.0000341	-37.1482	0.0000
Parameter 3	$\mathbf{6.3 * 10^{-6}}$	$\mathbf{2.848\ 10^{-7}}$	19.2603	0.0000

These findings collectively indicate the efficacy and reliability of the empirical model. The performance of these charts can be assessed using the Average Run Length (ARL), as indicated by Equation 10 presented below:

$$ARL = \frac{1}{1-\Phi(K-\delta\sqrt{n})\sqrt{n}+\emptyset(-K-\delta\sqrt{n})} \tag{10}$$

Where Φ represents the cumulative distribution of the standard normal, K is the number of standard deviations, n is the sample size, and δ is the shift in the mean of the residuals when the process exhibits a new source of variability, altering the regression function profile. Additionally, we can develop new equations for the ARL considering the estimation errors of the mean of Y, $\hat{\mu}_{(Y|x_0)}$, for a given value of X, and the estimation error of the residual variance. Equation 11 refers to the second case (UK), as follows:

$$ARL = \int_{-\infty}^{\infty} \frac{1}{\left[1-\Phi\left(\left[\frac{W}{\sqrt{m}}+\delta\sqrt{n}+K\right]\right)+\Phi\left(\left[\frac{W}{\sqrt{m}}-\delta\sqrt{n}-K\right]\right)\right]}\varphi(w)dw \tag{11}$$

Where φ(w) is the probability density distribution of the standard normal, $W \sim N(\mu = 0;\ \sigma = 1)$, m is the number of samples of size n. In this case, the control limits are determined, and a process will be under control when $e_i \in \left(\bar{e} - K\frac{\sigma_e}{m}; \bar{e} + K\frac{\sigma_e}{m}\right) = \left(\widehat{LCL}; \widehat{UCL}\right)$ where $\bar{e}$ is the mean of the residuals obtained by:

$$\bar{e} = \frac{1}{m}\sum_{i=1}^{m} e_i \tag{12}$$

Given that m is the number of samples of size n. Considering that σ_e is estimated in the calculation of the control limits of the residuals (case KU), we can arrive at the following equation for the ARL, assuming that $e_i \sim N(0, \sigma_e)$:

$$ARL = \int_{0}^{\infty} \frac{1}{1-\Phi(\boldsymbol{KU-\delta\sqrt{m}})+\emptyset(\boldsymbol{-KU-\delta\sqrt{m}})} f(u)du \tag{12}$$

Where $f(u) = (m-p)f_{Y^2}(m-p)u)$ represents the unconditional average run length, and f_{Y^2} denotes the probability density function of the Chi-Squared distribution with $m(n-p)$ degrees of freedom.

Finally, considering that estimates of the mean of Y and the standard deviation (Case UU) of the residuals are also estimated, we have the following equations for the ARL in this case: $P\left(-\frac{W}{\sqrt{m}} - KU - \delta\sqrt{n} \leq Z \leq \frac{W}{\sqrt{m}} + KU - \delta\sqrt{n}\right)$, where $W = \frac{\sqrt{m}(\bar{e}-\mu_0)}{\sigma_e}$, it follows a n (0,1) normal distribution, and the probability function of U is $f(u) = m(n-p)f_{Y^2}m(n-p)u)$. Thus, the unconditional average run length is the first moment (expected value) of the non-geometric unconditional run length (Jardim *et al.*, 2020) presented in Equation 13 with the probability $p(W, U, K, \delta, m)$, it can be stated that the average run length ARL is given by:

$$ARL = \int_{-\infty}^{\infty}\int_{0}^{\infty} \frac{1}{1-\Phi\left(\frac{W}{\sqrt{m}}+KU-\delta\sqrt{n}\right)+\emptyset\left(-\frac{W}{\sqrt{m}}-KU-\delta\sqrt{n}\right)} f(u)\varphi(w)dudw \tag{13}$$

Where φ denotes the pdf of a $N(\mu = 0;\ \sigma = 1)$ random variable and $f_{Y^2}(y)$ is the density function of a chi-square distribution with $m(n-p)$ degrees of freedom.

The Equation 13 provides a means to assess the effectiveness of residual control chart performance across varying parameters, including the number of curves derived from experimental data (m), the sample size (n), and δ, which represents the number of standard deviations from the mean of the residual. While theoretically, the average residual is expected to be zero with a standard deviation of sigma, the polynomial curve serves as an estimation, suggesting that the average residual will approximate zero if the model is appropriately adjusted.

Analysis of Table 2 reveals the Average Run Length (ARL) results, which offer insights into the performance of the residual control chart. The table showcases the ARL values under different parameter settings, demonstrating the impact of m, n, and δ on the chart's sensitivity to detecting process deviations. When $\delta = 0$, the process is considered to be in control. Conversely, when $\delta > 0$, the process is deemed to be out of control. Accordingly, the values presented in Table 2 represent the in-control Average Run Length (ARL) for $\delta = 0$ and the out-of-control ARL for $\delta > 0$. It is important to note that higher ARL values indicate a lower probability of detecting an out-of-control condition prematurely, while lower ARL values reflect a greater sensitivity of the chart in identifying process shifts. For example, when m = 1, n = 50, and δ = 0, the ARL is 16,610, meaning that, on average, 16,610 samples would be required to signal a change when no actual shift is present. As δ increases, the ARL values decrease, highlighting the increased responsiveness of the control chart to deviations in the process.

These results underscore the critical role of parameter selection in enhancing the effectiveness of residual control charts for accurate process monitoring and quality assurance.

Table 2 ARL for residual control chart with K=3

m	n	$\delta = 0$	$\delta = 0.25$	$\delta = 0.5$	$\delta = 0.75$	$\delta = 1.0$
1	50	16610	3972.	62.10	1.208	1.006
1	100	694.9	52.62	1.203	1.001	1.000
1	150	334.6	9.167	1.017	1.000	1.000
2	50	1137.	112.9	1.056	1.000	1.000
2	100	355.8	9.495	1.078	1.000	1.000
2	150	274.7	3.292	1.007	1.000	1.000
3	150	276.3	2.605	1.004	1.000	1.000
4	150	283.2	2.380	1.003	1.000	1.000
8	150	308.5	2.125	1.002	1.000	1.000
10	150	317.4	2.083	1.002	1.000	1.000

Table 3 presents the performance of the proposed control chart based on Equation 13. The results report the Average Run Length (ARL) values under varying conditions, considering the number of samples (m = 1 or 2), sample sizes (n = 50, 100, 115, and 150), and different levels of mean shift (δ = 0.0, 0.25, 0.50 and 1.0). In this analysis, the statistical parameters were estimated using Equation 13, where the average value of each response variable Y was modeled as a function of the explanatory variable X, along with the corresponding residual standard deviation. When estimating residuals from a single profile (m = 1), the sample size must exceed 150 observations to reliably detect a mean shift of δ = 0.5. This requirement highlights a limitation in sensitivity when only one curve is used. However, when m = 2—meaning two profiles (or two observations of Y for each X)—the chart demonstrates improved sensitivity to small shifts in the mean. This indicates an enhanced ability to detect subtle process changes, reinforcing the benefit of multiple observations for more effective monitoring.

Overall, the findings emphasize the importance of both the number of profiles and the sample size in determining the detection capability of the residual control chart. Increasing m contributes to more robust monitoring, particularly for identifying moderate to small process shifts.

Table 3 - ARL of Profile Control Chart

m	n	k	$\delta = 0$	$\delta = 0.25$	$\delta = 0.5$	$\delta = 1.0$
1	50	3	11144	2478.3	4.302	1.000
	100		587.59	19.408	1.667	1.000
	150		307.11	4.3390	1.000	1.000
	115		440.61	10.601	1.012	1.000
2	50		798.52	50.609	1.827	1.000
	100		315.52	5.8914	1.011	1.000
	150		255.10	2.5412	1.000	1.000
	115		289.24	4.2842	1.000	1.000

4.2 Curve of tool wear rate

Monitoring and controlling the variability of unidentified sources is recommended in control charts. In the case of Figure 1, we propose the control of standardized or non-standardized residuals together with monitoring the profile whose curve was fitted to the data by a second-order polynomial. However, it should be added that another characteristic related to the process efficiency is the tool wear rate, whose effect is measured by the first derivative of the fitted model. The derivative is given by the limit ($\lim_{x \to x_0} \frac{f(x)-f(x_0)}{x-x_0}$) , where Equation 14 is as follows:

$$d = -0.126689 * 10^{-2} + 0.19971 * 10^{-4} v \quad (14)$$

Where v indicates the production sequence and d represents tool wear rate. Figure 4 shows the consumption rate (in mm) and the limits determined from the standard deviation of the random error. It is observed that the tool wear rate is decreasing, with a higher rate in the production of the first piece. The purpose of this chart is to monitor process wear and establish a reference standard for resource, equipment, and machinery usage improvement projects.

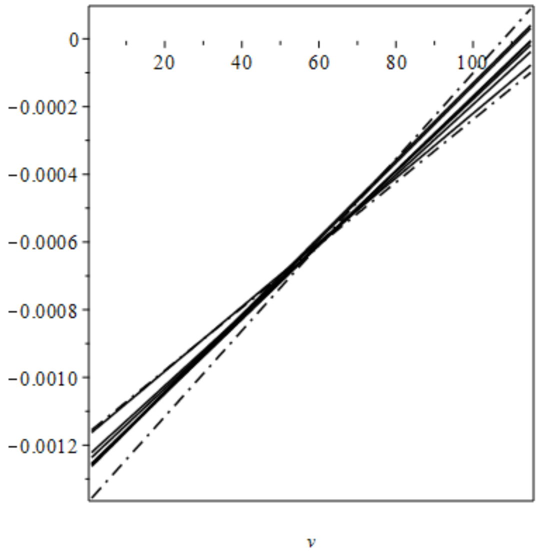

Figure 4 - Curve of tools wear rate (millimeter by piece).

In order to determine the stability point of the fitted function, we derived the function and set it equal to zero, finding the minimum point at 115, which is the number of sampled pieces, indicating the need to stop the machine for adjustments. Considering the estimation errors of the polynomial parameters, the machine adjustment moment could occur between 108 to 126 pieces, as shown in Figure 4. These results were obtained from the standard errors of the parameter estimates for a 99% confidence interval. The functions fit the curves data from figure 4 are:

$$1. Center\ Line = -0.0012669 + 0.000011 * j \quad (15)$$

$$2. Lower\ Line = -0.001165 + 0.00000093 * j \quad (16)$$

$$3. Upper\ Line = -0.001369 + 0.000013 * j \quad (17)$$

Where j represents the production sequence of pieces. In Figure 4, we have shown the simulation for curves with different parameter estimates of the mathematical model fitted by tool wear rate. The change in rate could indicate the presence of a special cause in the process. Therefore, we could use these tools to monitor the process along with residual and profile control charts.

4.3 Modified control chart applied in profile

According to Montgomery (2019) and Holmes and Mergen (2000), there is processes that, due to their nature, exhibit inevitable changes in the mean value of the quality characteristic of interest but still are capable of meeting the established specifications. This situation occurs when the process standard deviation is very small compared to the width of the tolerance (i.e., the difference between the upper and lower specification limits). In terms of standard statistical process control, this process, although not necessarily in control, is capable of producing acceptable products that must be protected against rejection. Montgomery (2019) argues that, in achieving a high level of process capability, it is sometimes useful to relax the level of surveillance provided by standard control charts.

Considering that, by observing Figures 1 and 3 and comparing the measurements with the specification (Ø65.60+0.2), whose tolerance is 0.2 tenths of a millimeter, the process is highly capable. In this case, the estimated standard deviation of the residual was $\hat{\sigma}_e = 0.003\ mm$. With the aim of maximizing the use of tools, given that their wear occurs over time, the operational procedure is to adjust the machining process equipment close to the upper engineering specification limit. The proposal is to include acceptance control limits in the profile chart to parameterize the cyclical process adjustments. The basic concept behind the first approach, the modified ¯X chart, is to allow the process mean to shift in such a way that the fraction of nonconforming pieces produced does not exceed a specified value δ. Freund (1957) provide a general discussion of this technique. Montgomery (2019) also provides an extensive reference on this technique from the statistical theory perspective.

As mentioned earlier, it is considered that in the present case the quality characteristic is normally distributed with a mean $\mu_{(y|x)}$ and a variance of σ_e^2. For a process with bilateral specification limits, in order to produce pieces with a nonconforming fraction lower than δ, the process mean μ can only shift within de μ_L and μ_U, as shown in Figure 5.

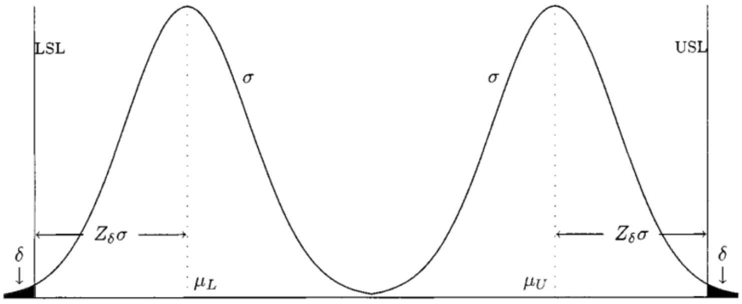

Figure 5 - Distribution of normal quality characteristic. Source: Adapted of Chang e Gan (1999)

Thus, the control limits are obtained by:

$$LCL = LSL + \left(Z_\gamma - \frac{Z_{\alpha/2}}{\sqrt{n}}\right)\sigma_e \tag{18}$$

$$UCL = USL - \left(Z_\gamma - \frac{Z_{\alpha/2}}{\sqrt{n}}\right)\sigma_e \tag{19}$$

Where γ represents a fraction of nonconforming items, α the Type I error, n is the sample size for the mean estimation, Z the standard normal percentile, Lower Specification Limits (LSL), and Upper Specification Limits (USL). Given that the machine is adjusted from the upper specification limit, and for a minimum Cpk, as a customer requirement, it is suggested that the process be adjusted by the value given in Equation 20. How $Cpk = min\left(\frac{USL-\mu_{(y|x)}}{3\sigma_e}, \frac{\mu_{(y|x)}-LSL}{3\sigma_e}\right)$, we have obtained following equations:

$$\mu_{(y|x)} = LSL + Cpk.(3\sigma_e) \tag{20}$$

$$\mu_{(y|x)} = USL - Cpk.(3\sigma_e) \tag{21}$$

For case, taking $Cpk = 1.67$ we have γ of 0.27 ppm (parts per million) with $Z_\gamma = 5.0$, $\hat{\sigma}_e = 0.003$, and USL=65.80 and LSL=65.60, using of Equation 20 and 21 we have the minimum and maximum limits value to $\mu_{(y|x)}$: $LCL = 65{,}62$, $UCL = 65{,}78$. The adjustment and regulation mechanism for this specific process is based on the UCL and LCL. Figure 6 shows how the proposed chart would look with the modified control limits in some simulated cases.

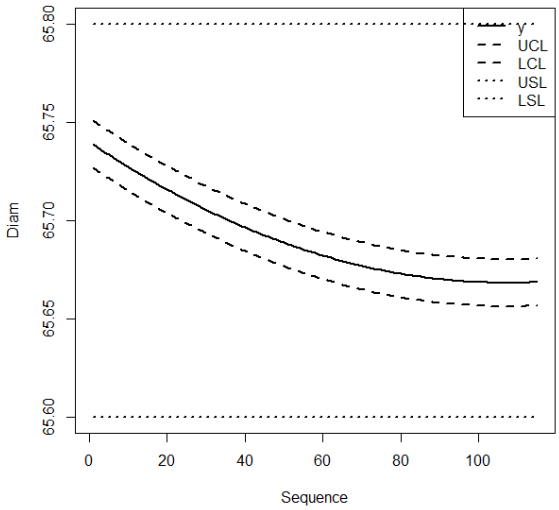

Figure 6 - Control limits for modified and profile control char.

4.4 Exploring theoretical and practical implications of the developed study

A challenge for the process owner described so far is to determine the process capability indices, Cp and Cpk. Explaining these capability indices using traditional concepts is not possible, as the quality characteristic Y does not follow a normal distribution, and the process is subject to mean variation over time. Nemati Keshteli *et al.* (2014), at the time, pointed out that there were few articles on process capability indices in profiles, and most of them focused on process capability indices applied to linear profiles. Since then, several studies have been conducted to propose process capability indices for profiles with univariate and multivariate response data (Maleki *et al.*, 2017). Nemati Keshteli *et al.* (2014) proposed calculating Cp and Cpk indices based on traditional methods in univariate models. We propose to calculate Cp based on the standard deviation of the residual, and Cpk at the extremes of the profile, especially at the beginning of production, where the equipment is adjusted under conditions that maximize the efficient use of production resources. Thus, we propose the following calculations for Cp and Cpk using Equations 20 and 21, with Equation 23 derived from Equation 21.

$$\hat{C}_p = \frac{USL-LSL}{6\hat{\sigma}_e} \quad (22)$$

$$\hat{C}_{pk} = \frac{USL-\hat{\mu}(y|x_0)}{3\hat{\sigma}_e} \quad (23)$$

Where x_0 represents the value of the predictor variable in the worst condition that has the highest probability of producing out-of-specification parts; in this case, x_1, which is the first piece produced and where it represents the value of the mean of Y that maximizes the use of the tooling. For the present case, we can consider that the random variability is estimated by the standard deviation of the residual. Thus, the potential capability, obtained by Equation 23, for the process is 11.1, which is well above the 1.67 recommended for critical characteristics. If we adjust the process under the conditions predicted in Equations 22 and 23, for a *Cpk*=1.67, the process should be adjusted so that the mean of Y is 65.78 and 65.62 mm.

The development provided elements for the construction of a method for monitoring processes that exhibit some effect of linear trend, as is the case of the example presented here. It is necessary to consider that these processes with this characteristic have a normal distribution of residuals, possess high capability, and fit a trend curve.

There are indications in the literature on how to proceed in these cases; for example, Montgomery (2019) suggests monitoring the coefficients of the regression model, as well as monitoring the residuals through a variance chart or standardized residuals. Recent articles explore new approaches for monitoring processes with linear regression or profiles. Abbas *et al.* (2023) propose auxiliary information-based (AIB) control charts, which have been shown to be efficient for early detection of changes in process parameters. Regarding profile charts, Woodall and Montgomery (2014), Noorossana *et al.* (2011), and Woodall *et al.* (2004) provide an overview of some of the general issues involved in using control charts to monitor linear and nonlinear profiles.

Shper and Adler (2017) address the use of control charts in situations similar to the example presented. The case studied in this article clearly exhibits a temporal trend due to identifiable causes, however, acceptable for process performance. The first step in implementing statistical monitoring in situations with linear or nonlinear trends is to understand and adjust a trend curve model. Thus, it is possible to predict the variable of interest as a function of time or an independent variable. The importance of understanding the temporal behavior of the process characteristic is indicated in the literature by Shper and Adler (2017).

The application of support variables through linear regression models, another topic highlighted in the literature, reinforces the need to monitor the variability of random sources of unidentified causes, called residuals in regression analysis. Monitoring the curve profile along with the residuals are complementary methods and recommended in the classical Shewhart approach. However, for better prediction of results, we include in the development of the artifacts the use of the first derivative for two purposes: i) anticipate changes in the curve profile due to identifiable causes, ii) identify minimum or maximum points of the curve, which can be useful in evaluating overall process performance.

Another monitoring tool in practical situations is the use of location control charts and modified charts, which determine the time of equipment and tool adjustment, in order to ensure minimum acceptable capability, as well as maximizing the use of these resources.

5 CONCLUSION

The artifacts developed from a real-world problem, which involved the application, modeling, and performance evaluation of control charts in processes with linear trend effects, aimed to contribute to the development of the field of knowledge in statistical monitoring, focusing on practical issues. We innovated in this work by fully applying widely known methods, consolidating a comprehensive approach to the problem that included the use of profile control charts, monitoring of regression model parameters, application of the location control chart approach, acceptance charts, and capability calculations in profiles. We analytically assessed the performance of these charts through ARL, and through these analyses, we indicated the effects of sample sizes used in phase I to estimate the parameters of the regression model on performance in phase II.

This article proposes an innovative approach to monitoring processes with temporal trends, using profile curves and statistical modeling based on location and residual control charts. By using prescriptive decision models and the DSR approach, we sought to solve practical problems and contribute to the advancement of knowledge in statistical process monitoring. The results have demonstrated that processes with linear trends can be adequately monitored and controlled, even when exhibiting non-random behavior over time. The study also offers a theoretical and practical framework for the implementation of these techniques. This artifact contributes with a more effective approach to identifying and managing changes in industrial processes. Finally, the concluding remarks emphasize the importance of integrated and adaptable approaches to process monitoring, with an emphasis on the application of advanced statistical methods and a detailed understanding of the characteristics of the process at hand.

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Contributions for authors: PCO: Conceptualization; Methodology; Validation; Software; Formal analysis; Investigation; Data Curation; Resources; Writing - Original Draft; Writing - Review & Editing; Visualization; Supervision; Project administration; Funding acquisition. DCV: Conceptualization; Methodology; Validation; Formal analysis; Visualization; Data Curation; Resources; Writing - Review & Editing; Funding acquisition. JEC: Conceptualization; Validation; Data Curation; Writing - Review & Editing; Visualization.