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RESEARCH PAPER

Profit Optimization for a Manufacturing Supply Chain Under Carbon Emission and with Inventory - Price Based Demand

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ABSTRACT

Highlights: A manufacturing supply chain dealing with decaying inventory is studied with three production sub-cycles. Price-based demand, the Weibull degradation rate and carbon tax policy are taken into account. Price, carbon emission factor, cycle length and production period are decision variables to increase profit.

Goal: The research work aims to maximize profit for a manufacturing supply chain dealing with deteriorating inventories. To maximize profit, the optimum value of production cycle length, total cycle length, price, and emission factor were studied.

Design/ Methodology/ Approach: A manufacturing supply chain is studied in order to maximize profit. All assumptions are mathematically stated as a profit function model. Using a numerical problem, the analysis and practical implications of the projected optimization issue have been demonstrated. Finally, an efficient conclusion is constructed to demonstrate the applicability of the optimization profit model on the basis of sensitivity analysis.

Results: It is concluded to maximize profit, cycle length, price and carbon emission need to be minimize and maximize production length. It is also concluded that as the price increases demand is decreases that decreases profit.

Limitations of the investigation: The profit model is designed under certain assumptions like deterioration rate, demand rate, number cycles, so the model is applicable for such conditions.

Practical implications: This research work may help to manage the manufacturing supply chain for deteriorating inventories to maximize profit with minimum carbon emission.

Originality / values: The present research work is motivated by the problems of the manufacturers and organizations dealing with deteriorating inventories.

Key words: Weibull deterioration; Manufacturing; Carbon emission; Price–inventory-based demand.

1. INTRODUCTION

Green environment and emission reduction are new major problems all around the world at the moment. Today, a lot of groups are attempting to create a greener environment and effective policies to balance it. As we all know, businesses are also accountable for carbon emissions, therefore they are compelled to follow the regulations created by academics and policymakers, as well as the government, who are all aiming to minimize emissions and improve societal welfare. These policies include a carbon tax, a carbon offset, and a carbon cap. One of these measures is the carbon tax, which imposes a set tax on each unit of emission produced.The implementation of these rules forces businesses to adapt their strategy in order to increase profit while limiting emissions. It is difficult for businesses to strike a balance between emission reductions and

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profitability.

In other words, it is a balance between government policy and supply chain profitability. As a result, each manufacturing supply chain that deals with degrading inventory currently works for two goals: the primary goal is "how to increase profits," and the second is "how to limit emission." The goal of the presented study is to increase profits by selecting the best values for the decision variables total cycle duration, production period, price, and emission factor. Here, we apply a carbon tax policy to reduce carbon emissions, allowing us to meet the core supply chain objectives of reducing carbon emissions while increasing profit. Minimum carbon emissions are also a focus of the supply chain with the highest profit.Therefore, in presenting research work above mentioned two goals are main objectives. Decay or degradation is a process in which any usable thing changes its functional values, and this value tends to zero after some time due to any factor such as weather, temperature, poor stocking and transportation, and so on. Aside from natural degradation, there is another type of deterioration of fashion goods (mobile phones, fashion items, vehicles, and so on) in which natural features stay constant but the products are rejected owing to fashion, quicker technology, and alternatives. The rate of degradation is affected by weather, temperature, warehouse location, and a variety of other external and internal factors.

In our research, we used a three-parameter Weibull distribution to determine the decay rate. For goods whose rate of degradation rises with time, the Weibull distribution is more practical and versatile. Mathematical modelling with the Weibull distribution for rate of decay provides greater model flexibility since it may be used with any sort of rate of decay. Many commodities begin the degradation process right once, while certain products have a shelf life. As a result, we adopted the Weibull distribution so that this model may be applied for any decaying natural inventory. We chose a three parameter weibull distribution over a two parameter weibull distribution because it is often insufficient to illustrate the nature of the fading process. The two parameter weibull distribution works only for objects with either a growing or decreasing rate of deterioration, which means the item has an initial rate of deterioration that is either extremely high or very near to zero, however this is not true for all types of decaying products. As a result, the three parameter Weibull distribution is more practical. Therefore we used three parameter Weibull distribution for deterioration rate. Figure 1 depicts a three parameter Weibull distribution for the rate of degradation function of three parameters. Aside from the two parameters of the Weibull distribution, there is one more parameter that stands for the self life of an item; this is a significant property of most decaying objects.

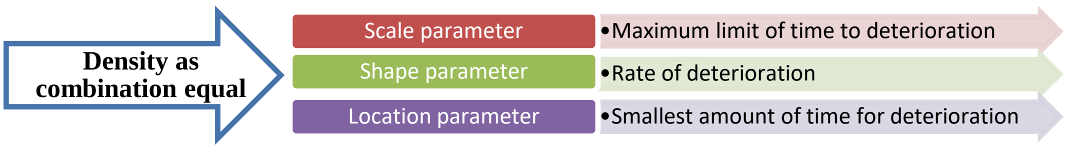

Figure 1

2. LITERATURE REVIEW

(Chakrabarty et al., 1998) studied deteriorating inventory supply chain with linear demand and three-parameter Weibull distribution for deterioration along with shortages. (He et al., 2010) studied how to take advantage of the gaps in the timing of the selling season between geographically discrete markets for deteriorating inventories for increasing the profitability of the firm. (Kouki et al., 2016) established a model for multiple deteriorating items considering random life cycles by using Markov process. (Grillo et al., 2017) studied a complex order promising process for fruit supply chains by establishing mixed-integer mathematical modeling. (Mubiru, 2018) studied a drugs supply chain with Stochastic demand with replenishment policy. (Daryanto et al., 2019) considered three echelon supply chain with carbon emission generation due to transportation, deterioration and warehousing with the objective of finding number of order and size of order. (Frazzon et al., 2019) this work is evolution of supply chain management means this paper studied about how supply management changes with time and new technologies. (Sazvar and Sepehri, 2019) discussed and relate the deteriorating inventory supply chain with economic, environmental, and social issues by using data from the flower industry. They developed mathematical modelling for a deteriorating inventory supply chain that create job for the local workforce. (Liu et al., 2019) discussed the coordinated system between seller and buyer and developed a policy for production and shipment to minimize the joint total cost. (Dehghani et al., 2019) worked for a blood supply chain to balance the shortage and wastage of blood units.

(Jing and Mu, 2019) studied a dynamic lot-sizing problem with two types of problems one for stock deterioration rates depending upon the age of the inventory and the second is considering the substitution of a product with any other product that is one way. (Otrodi et al., 2019) discussed a lot-sizing problem with multiple demands and two-level trade credit policy. (Yang et al., 2019) studied the cross effect of decaying and suggest a global mixed perishable inventory policy so as to reduce deterioration cost.

(Wang et al., 2019) studied deteriorating inventory supply chain with different replenishment policies such as separate replenishment, joint replenishment dealing with the large-scale supplier and multiple. They have also considered a carbon cap-and-trade policy. (Bhatia et al., 2020) studied manufacturing supply chain with the enviromental management objective. (Huang et al., 2020) worked for inventory management supply chain with carbon control policies and with green investment. (Paraschos et al., 2020) studied a supply chain that produces a single type of product with a stochastic production inventory system under reinforcement learning. (Yang et al., 2020) defined the deterioration rate as one of the controllable factors. (Khakzad and Gholamian, 2020), studied the effect of inspection during a supply chain of deteriorating inventory supply chain. (Ghiami and Beullens et al., 2020) studied warehouse policy with stock-dependent demand in which the retailer looks for an integrated optimal distribution policy alliance with a supplier.

(Yu et al., 2020) studied the deteriorating inventory supply chain with demand as a function of price-inventory. They have also focused on the green supply chain and carbon emission during the supply chain and established a model with preservation technology. (Wu and Kung, 2020) works for different carbon policies and studied that use of clean energy minimizes the market competitiveness of firms. (Sepehri and Gholamian, 2021) studied deteriorating inventory supply chain with preservation techniques to control emissions. (Abdul Halim et al., 2021) discussed a production opening inventory model for deteriorating items under overtime concept with price and available inventory focused demand. (Mishra et al., 2019b) also established mathematical modeling for deteriorating inventor supply chain in which demand is function of stock and price both.

As in present time, green supply chain with profitability both is prior need of any business firm. Therefore, many researchers and policymakers contributed to the green supply chain. The government established a few policies for a green supply chain with the motive to encourage social welfare. Business organizations are trying to establish a balance between two objectives of minimizing carbon emissions and maximizing the profit of the firm. Some of recent works are as follows. (Halat and Hafezalkotob, 2019) developed mathematical modeling for a multi-stage supply chain considering carbon emission regulation policy by using the game theory approach. (Mishra et al., 2019a) and (Mishra et al., 2019b) studied a supply chain for deteriorating inventories with the objective of minimize carbon generation. (Wang et al., 2019) studied carbon cap-and-trade policy for deteriorating inventory supply chain with different replenishment policies such as separate replenishment, joint replenishment dealing with the large-scale supplier, and multiple. (Bhatia et al., 2020) studied a closed-loop supply chain for the management of carbon emissions. (Huang et al., 2020), observes the combined effect of emission regulation policies and green technologies on the two-echelon supply chain. (Wu and Kung, 2020) established two models (asymmetric duopoly) considering different carbon emission technologies. (Yu et al., 2020) studied deteriorating inventory supply chain with consideration that ordering and stocking of decaying inventories produces carbon emission and established a model with preservation technology. (Mishra et al., 2021), explored about sustainable inventory management under controllable carbon emissions from a greenhouse farm. (Sepehri and Gholamian, 2021), studied carbon emissions due to deteriorating inventory and used preservation techniques to control emission with discrete values of the cycle length. The above given literature review based on the key words and objective of our paper. Presenting paper considered a manufacturing supply chain price-stock based demand, weibull deterioration rate (three parameter) with carbon tax policy to minimize carbon emission. The objective of the paper is to maximize profit. We couldn't locate any studies that included price-stock-based demand, the Weibull degradation rate (three parameters), and a carbon tax policy in a manufacturing supply chain.

2.1 Discussion section

Following a review of the literature, we conclude that the major goal of any manufacturing supply chain, which is producing degrading nature stocks, is to meet both the objectives of profit and minimizing carbon emissions. There are very few studies that tackle both objectives simultaneously in a manufacturing supply chain. We couldn't locate any studies that included price-stock-based demand, the Weibull degradation rate (three parameters), and a carbon tax policy in a manufacturing supply chain. The study's purpose is to maximize profits by selecting the optimum values for decision variables such as total cycle duration, production period, pricing, and emission factor. Here,we use a carbon tax policy to reduce carbon emissions, allowing us to accomplish the primary supply chain objectives of lowering carbon emissions while boosting profit. As a result, the research activity has two goals: to reduce the carbon factor and to optimize profit.

3. ASSUMPTIONS

Assumptions implied to illustrate the mathematical modeling are explained here briefly.

1. Here it is assumed that the during one cycle the manufacturing takes place in three small cycles that termed as sub cycles and considered as $(0,t_1)$, (t_1,t_2), (t_2,t_3), (t_3,t_4), (t_4,t_5) and (t_5,T) are all subset of $(0,T)$.Here in sub cycles $(0,t_1)$, (t_1,t_2), (t_2,t_3) manufacturing will be takes place and inventory decreases due to demand and deterioration. During sub cycle (t_3,t_4) no manufacturing, (t_4,t_5) no manufacturing and shortages occur and during sub cycle (t_5,T) shortages will be fulfilled first.

2. The demand rate is price and available inventory dependent which is described as follows: $D(p,I(t)) = D(p) + \delta(I(t))$ where $0 < \delta << 1$ and $D(p) = \tau(x_1 - yp) + (1-\tau)x_2 p^{-\mu}$ is a hybrid demand both for linear and nonlinear price which satisfying following condition. $0 \le \tau \le 1$, $x_1 > 0,\ x_2 > 0,\ y > 0,\ \frac{x_1}{y} > p$ and $\mu > 1$.We picked up such type of demand from (Mishra et al., 20190).

3. Deterioration directly proportion to time and density of deterioration is function of three parameter Weibull rate. The time distributions for decay of the inventory is given by $\sigma(t) = \alpha\beta(t-\gamma)^{\beta-1}e^{-\alpha(t-\gamma)^{\beta}}; t > 0$; here α, β and γ are scale, shape and location parameters respectively with the conditions such as $0 < \alpha \le 1$; $\beta > 1$, here we assume $\beta = 2$ and $\gamma > 0$ and $t \ge \gamma$. The application of the three-parameter Weibull deterioration rate is useful as it helps to recognize the already decayed inventories received in the supply chain and it also estimate the inventories that may be decay in future.

4. The time horizon of the production cycle system is finite.

5. Shortages are permissible and entirely backlogged.

6. Carbon emission factor taken into consideration during disposal of deteriorating inventories.

4. REPRESENTATIVE NOTATION

D	Demand in units/unit time that is price and inventory based
P	Production rate in units/unit time, here $D < P$
$Q(t)$	Inventory level at time t
S_b	Maximum Shortage level of the product
α	Scale parameter where α >0
β	Shape parameter where β >0 also known as Weibull gradient and it is dimensionless. In present work β = 2 considered.
γ	γ is location parameter and its dimension same as time and γ >0
$\sigma(t)$	Density distribution function for deterioration where $\sigma(t) = \alpha\beta(t-\gamma)^{\beta-1}e^{-\alpha(t-\gamma)^{\beta}}; t > 0$
δ	Fraction of whole inventory where $0 < \delta < 1$
$\phi(t)$	Instant rate of deterioration
$\eta(t)$	Cumulative distribution function for three- parameter Weibull distribution
P_c	Manufacturing cost/unit time
H_c	Holding cost/time
S_P	Set cost/time
S_c	Shortage cost
a	Constant (Fraction of available inventory in $t_1 < t \le t_2$)

b	Constant (Fraction of available inventory in $t_2 < t \le t_3$)
$k_i, i = 1,2,3$	Constant multiple of sub cycle length
χ	Carbon emission tax per unit
PC	Production cost/time
SP	Set up cost/time
DC	Deterioration /time
HC	Holding cost/time
SC	Shortages cost/time
EC	Emission cost/ time
RC	Earn revenue/time
Decision variables	
T	Total Cycle length
t_i	Sub cycles length $(i = 1,2,3,4,5)$
v	Carbon emission per unit
p	Price/unit

5. MATHEMATICAL CALCULATION FOR DESIGNING AN EFFECTIVE MODEL THAT MAXIMIZE PROFIT FOR DISCUSSED TYPE OF MANUFACTURING SUPPLY CHAIN

An organized mathematical calculation for designing an effective model that maximize profit for discussed type of manufacturing supply chain has been carry out. For this it is assumed that production cycle length time is from $t = 0$ - $t = T$ with sub cycles $(0,t_1)$, (t_1,t_2), (t_2,t_3), (t_3,t_4), (t_4,t_5) and (t_5,T). During each sub cycle the inventory level has changed with $(P-D)$, $a(P-D)$ and $b(P-D)$ respectively where a and b are constants. Manufacturing work will be done in the sub cycles $(0,t_1)$, (t_1,t_2) and (t_2,t_3) and between (t_3,t_4) cycle production will not take place. Level of inventories decreases due to demand and deterioration during sub cycles $(0,t_1)$, (t_1,t_2), (t_2,t_3) and (t_3,t_4) and during (t_4,t_5) shortages occur that are entirely backlogged. Here a designed mathematical modeling has been carried out.

Here, we have considered demand rate which is function of both price and available inventory in stock. which is explained in equations (1) , (2) and (3) as follows.

$$D(p,Q(t)) = D(p) + \delta(Q(t)) \text{, where } 0 < \delta << 1 \tag{1}$$

Here $$D(p) = \tau(x_1 - yp) + (1-\tau)x_2 p^{-\mu} \tag{2}$$

Where $0 \le \tau \le 1,\ x_1 > 0,\ x_2 > 0,\ y > 0,\ \dfrac{x_1}{y} > p$ and $\mu > 1$

By using equation (1) and (2), we get

$$D(p,Q(t)) = \tau(x_1 - yp) + (1-\tau)x_2 p^{-\mu} + \delta(Q(t)) \tag{3}$$

Above mention equations (1) , (2) and (3) are considered according to the work of (Mishra et al., 2019).

Now allocation of density for rate of deterioration used by (Chakrabarty et al., 1998) is given by

$$\sigma(t) = \alpha\beta(t-\gamma)^{\beta-1} e^{-\alpha(t-\gamma)^{\beta}} ; t > 0 \tag{4}$$

Let's assume that the decaying rate of manufactured products be given by $\phi(t)$ at time t which is given by

$$\phi(t) = \frac{\sigma(t)}{1-\eta(t)}$$

Where $\eta(t)=1-e^{-\alpha(t-\gamma)^{\beta}}$ is the cumulative distribution function for three- parameter Weibull distribution by using equation (4) and (5)

$$\phi(t)=\alpha\beta(t-\gamma)^{\beta-1} \tag{6}$$

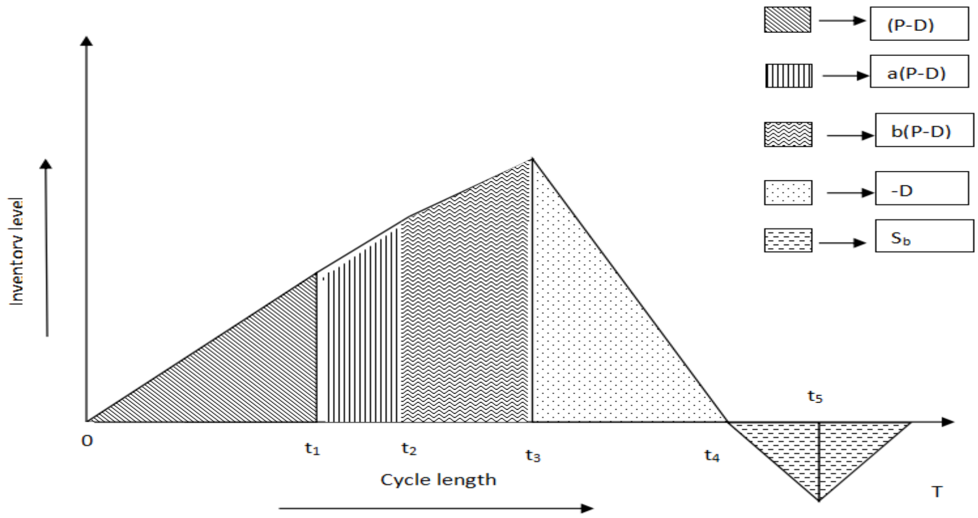

Figure 2 Inventory level in different sub cycles

Now by using above assumptions, notations and conditions of figure 2, the mathematical modeling for profit function for discussed case will be takes place.

$$\frac{dQ(t)}{dt}+\phi Q(t)=P-D(Q(t),p); \quad 0\le t\le t_1 \tag{7}$$

$$\frac{dQ(t)}{dt}+\phi Q(t)=P-\left(D(p)+\delta Q(t)\right); \quad 0\le t\le t_1$$

(8)

$$\frac{dQ(t)}{dt}+\phi Q(t)=a\left(P-D\left(p\right)+\delta Q(t)\right); \quad t_1<t\le t_2$$

(8)

$$\frac{dQ(t)}{dt}+\phi Q(t)=b\left(P-D\left(p\right)+\delta Q(t)\right); \quad t_2<t\le t_3 \tag{9}$$

$$\frac{dQ(t)}{dt}+\phi Q(t)=-\left(D\left(p\right)+\delta Q(t)\right); \quad t_3<t\le t_4 \tag{10}$$

$$\frac{dQ(t)}{dt}=-\left(D\left(p\right)+\delta Q(t)\right); \quad t_4<t\le t_5 \tag{11}$$

$$\frac{dQ(t)}{dt}=\left(P-\left(D\left(p\right)+\delta Q(t)\right)\right); \quad t_5<t\le T \tag{12}$$

To solve above given equations with $\phi(t)=\alpha\beta(t-\gamma)^{\beta-1}$ where $0<\alpha\le 1$, $\beta>1$ and $\gamma>0$ and the boundary conditions are that gives inventory level at different time period.

$$Q(0)=0,\ Q(t_4)=0,\ Q(t_5)=S_b \text{ and } Q(T)=0 \tag{13}$$

By solving equation (7), it is obtained

$$Q(t)=(P-D)\left\{t+\frac{\delta t^2}{2}+\frac{\alpha}{\beta+1}\left((t-\gamma)^{\beta+1}-(-\gamma)^{\beta+1}\right)\right\}e^{-\alpha(t-\gamma)^{\beta}}$$

By simplifying above equation and ignoring higher power of α as $\alpha<<1$, it is obtained

$$Q(t)=(P-D)\left\{t+\frac{\delta t^2}{2}+\frac{\alpha}{\beta+1}\left((t-\gamma)^{\beta+1}-(-\gamma)^{\beta+1}\right)-\left(t+\frac{\delta t^2}{2}\right)\alpha(t-\gamma)^{\beta}\right\} \quad (14)$$

Solution of equation (8) is, here also by ignoring higher power of α as $\alpha<<1$, it is obtained

$$Q(t)=a(p-D)\left\{t+\frac{\delta t^2}{2}+\frac{\alpha}{\beta+1}\left((t-\gamma)^{\beta+1}-(-\gamma)^{\beta+1}\right)-\left(t+\frac{\delta t^2}{2}\right)\alpha(t-\gamma)^{\beta}\right\} \quad (15)$$

Solution of equation (9) is, here also by ignoring higher power of α as $\alpha<<1$, it is obtained

$$Q(t)=b(p-D)\left\{t+\frac{\delta t^2}{2}+\frac{\alpha}{\beta+1}\left((t-\gamma)^{\beta+1}-(-\gamma)^{\beta+1}\right)-\left(t+\frac{\delta t^2}{2}\right)\alpha(t-\gamma)^{\beta}\right\} \quad (16)$$

Solution of equation (10) is, here also by ignoring higher power of α as $\alpha<<1$, it is obtained

$$Q(t)=D\left\{(t_4-t)+\frac{\alpha}{\beta+1}(t_4-\gamma)^{\beta+1}-(t-\gamma)^{\beta+1}+\left(\frac{\delta}{2}(t_4^2-t^2)\right)\right\}e^{-\alpha(t-\gamma)^{\beta}}$$

$$Q(t)=D(p)\left\{\begin{array}{l}(t_4-t)+\frac{\alpha}{\beta+1}\left((t_4-\gamma)^{\beta+1}-(t-\gamma)^{\beta+1}\right)+\left(\frac{\delta}{2}(t_4^2-t^2)\right)- \\ \alpha(t-\gamma)^{\beta}\left((t_4-t)+\left(\frac{\delta}{2}(t_4^2-t^2)\right)\right)\end{array}\right\} \quad (17)$$

Solution of equation (11) is, here also by ignoring higher power of α as $\alpha<<1$, it is obtained

$$Q(t)=D(p)\left((t_4-t)+\frac{\delta}{2}(t_4^2-t^2)+\delta t(t_4-t)\right) \quad (18)$$

Solution of equation (12) is, by ignoring higher power of α as $\alpha<<1$, it is obtained

$$Q(t)=-\left((P-D(p))\left((T-t)+\frac{\delta}{2}\left(T^2-t^2\right)+\delta t(T-t)\right)\right) \quad (19)$$

6. SHORTAGE LEVEL

Here it is discussed about shortages occurring in supply chain. By putting $t=t_5$ in equation (15) and use equation (13), it is obtained.

$$Q(t_5)=D(p)\left((t_4-t_5)+\frac{\delta}{2}(t_4^2-t_5^2)+\delta t_5(t_4-t_5)\right)=S_b$$

And by solving equation (16) and (10), we get

$$Q(t_5)=-\left((P-D(p))\left((T-t_5)+\frac{\delta}{2}\left(T^2-t_5^2\right)+\delta t_5(T-t_5)\right)\right)$$

Now equating these results we get (This result is calculated by Mathematica 0.9)

$$t_5=\frac{0.166667}{P\delta}\left(2\mathrm{D}-4\mathrm{P}-2DT\delta+2PT\delta+2Dt_4\delta\pm\sqrt{\begin{array}{l}(-2\mathrm{D}+4\mathrm{P}+2\mathrm{D}\,\mathrm{T}\delta-2\mathrm{PT}\delta-2\mathrm{D}\delta\mathrm{t}_4)^2- \\ 12\mathrm{P}\delta(4\mathrm{DT}-4\mathrm{PT}+\mathrm{DT}^2\delta-\mathrm{PT}^2\delta-2\mathrm{Dt}_4-\mathrm{D}\delta t_4^2)\end{array}}\right) \quad (20)$$

Here we get two values of t_5 in the form of a+b and a-b , for further calculation of our model we uses a-b because after observing results with both the values it is observed that a-b value gives us more appropriate value of decision variables and profit.

7. DIFFERENT COST FUNCTIONS AND EARN REVENUE

For calculating the total profit, first all costs occurring in supply chain need to be calculate by the following steps. They are subtracted together from total revenue after evaluating to find profit.

Cost of production

The manufacturing cost occurring in supply chain in cost/unit time is obtain by $PC = PP_c$ (21)

Set up cost /unit time SC $= S_p$ (22)

Holding cost/unit time

$$HC = \frac{H_c}{T}\left[\int_0^{t_1} Q(t)dt + \int_{t_1}^{t_2} Q(t)dt + \int_{t_2}^{t_3} Q(t)dt + \int_{t_3}^{t_4} Q(t)dt\right]$$

$$= \frac{H_c}{T}\left[\begin{array}{l}
\left(\int_0^{t_1}\left[(p-D)\left\{t+\frac{\delta t^2}{2}+\frac{\alpha}{\beta+1}\left((t-\gamma)^{\beta+1}-(-\gamma)^{\beta+1}\right)-\left(t+\frac{\delta t^2}{2}\right)\alpha(t-\gamma)^{\beta}\right\}\right]dt\right) \\
+\left(\int_{t_1}^{t_2}\left[a(p-D)\left\{t+\frac{\delta t^2}{2}+\frac{\alpha}{\beta+1}\left((t-\gamma)^{\beta+1}-(-\gamma)^{\beta+1}\right)-\left(t+\frac{\delta t^2}{2}\right)\alpha(t-\gamma)^{\beta}\right\}\right]dt\right) \\
+\left(\int_{t_2}^{t_3}\left[b(p-D)\left\{t+\frac{\delta t^2}{2}+\frac{\alpha}{\beta+1}\left((t-\gamma)^{\beta+1}-(-\gamma)^{\beta+1}\right)-\left(t+\frac{\delta t^2}{2}\right)\alpha(t-\gamma)^{\beta}\right\}\right]dt\right) \\
+\left(\int_{t_3}^{t_4}\left[D\left\{\begin{array}{l}(t_4-t)+\frac{\alpha}{\beta+1}\left((t_4-\gamma)^{\beta+1}-(t-\gamma)^{\beta+1}\right)+\left(\frac{\delta}{2}(t_4^2-t^2)\right)- \\ \alpha(t-\gamma)^{\beta}\left((t_4-t)+\left(\frac{\delta}{2}(t_4^2-t^2)\right)\right)\end{array}\right\}\right]dt\right)
\end{array}\right]$$

$$\frac{H_c}{T}\left[\begin{array}{l}\left[(p-D)\left\{\begin{array}{l}\frac{t_1^2}{2}+\frac{\delta t_1^3}{6}+\frac{\alpha}{\beta+1}\left(\left(\frac{(t_1-\gamma)^{\beta+2}-(-\gamma)^{\beta+2}}{(\beta+2)}\right)-t_1(-\gamma)^{\beta+1}\right)-\\ \alpha\left(\frac{t_1(t_1-\gamma)^{\beta+1}}{(\beta+1)}\right)+\alpha\left(\frac{(t_1-\gamma)^{\beta+2}-(-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)}\right)\end{array}\right\}\right]\\ +\left[a(p-D(p))\left\{\begin{array}{l}\left(\frac{t_2^2-t_1^2}{2}\right)+\frac{\delta\left(t_2^3-t_1^3\right)}{6}+\frac{\alpha}{\beta+1}\left(\begin{array}{l}\frac{\left(\left((t_2-\gamma)^{\beta+2}-(t_1-\gamma)^{\beta+2}\right)\right)}{\beta+2}-\\ \left(t_2-t_1\right)(-\gamma)^{\beta+1}\end{array}\right)\\ -\alpha\left(\frac{t_2(t_2-\gamma)^{\beta+1}-t_1(t_1-\gamma)^{\beta+1}}{(\beta+1)}-\frac{(t_2-\gamma)^{\beta+2}-(t_1-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)}\right)\end{array}\right\}\right]\\ +\left[b(p-D(p))\left\{\begin{array}{l}\left(\frac{t_3^2-t_2^2}{2}\right)+\frac{\delta\left(t_3^3-t_2^3\right)}{6}+\frac{\alpha}{\beta+1}\left(\begin{array}{l}\frac{\left(\left((t_3-\gamma)^{\beta+2}-(t_2-\gamma)^{\beta+2}\right)\right)}{\beta+2}-\\ \left(t_3-t_2\right)(-\gamma)^{\beta+1}\end{array}\right)\\ -\alpha\left(\frac{t_3(t_3-\gamma)^{\beta+1}-t_2(t_2-\gamma)^{\beta+1}}{(\beta+1)}-\frac{(t_3-\gamma)^{\beta+2}-(t_2-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)}\right)\end{array}\right\}\right]\\ +\left[D(p)\left\{\begin{array}{l}\left(\frac{(t_4-t_3)^2}{2}\right)+\frac{\alpha}{\beta+1}(t_4-\gamma)^{\beta+1}\left(t_4-t_3\right)-\left(\frac{(t_4-\gamma)^{\beta+2}-(t_3-\gamma)^{\beta+2}}{(\beta+2)}\right)+\\ \left(\frac{\delta}{2}\left(\frac{2t_4^3}{3}-t_3t_4^2+\frac{t_3^2}{2}\right)\right)+\alpha\left(\begin{array}{l}\left(\frac{(t_3-\gamma)^{\beta+1}(t_4-t_3)}{(\beta+1)}\right)+\\ \left(\frac{(t_4-\gamma)^{\beta+2}-(t_3-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)}\right)\end{array}\right)\end{array}\right\}\right]\end{array}\right] \tag{23}$$

Deteriorating cost/unit time

$$DC=\frac{P_c}{T}\left[\int_0^{t_1}\phi(t)Q(t)dt+\int_{t_1}^{t_2}\phi(t)Q(t)dt+\int_{t_2}^{t_3}\phi(t)Q(t)dt+\int_{t_3}^{t_4}\phi(t)Q(t)dt\right]$$

$$\frac{P_c}{T}\left[\begin{array}{l}\left(\int_0^{t_1}\alpha\beta(t-\gamma)^{\beta-1}\left[(P-D(p))\left\{t+\frac{\delta t^2}{2}+\frac{\alpha}{\beta+1}\left((t-\gamma)^{\beta+1}-(-\gamma)^{\beta+1}\right)-\left(t+\frac{\delta t^2}{2}\right)\alpha(t-\gamma)^{\beta}\right\}\right]dt\right)\\+\left(\int_{t_1}^{t_2}\alpha\beta(t-\gamma)^{\beta-1}\left[a(P-D(p))\left\{t+\frac{\delta t^2}{2}+\frac{\alpha}{\beta+1}\left((t-\gamma)^{\beta+1}-(-\gamma)^{\beta+1}\right)-\left(t+\frac{\delta t^2}{2}\right)\alpha(t-\gamma)^{\beta}\right\}\right]dt\right)\\+\left(\int_{t_2}^{t_3}\alpha\beta(t-\gamma)^{\beta-1}\left[b(P-D(p))\left\{t+\frac{\delta t^2}{2}+\frac{\alpha}{\beta+1}\left((t-\gamma)^{\beta+1}-(-\gamma)^{\beta+1}\right)-\left(t+\frac{\delta t^2}{2}\right)\alpha(t-\gamma)^{\beta}\right\}\right]dt\right)\\+\left(\int_{t_3}^{t_4}\alpha\beta(t-\gamma)^{\beta-1}\left[D(p)\left\{\begin{array}{l}(t_4-t)+\frac{\alpha}{\beta+1}\left((t_4-\gamma)^{\beta+1}-(t-\gamma)^{\beta+1}\right)+\left(\frac{\delta}{2}(t_4^2-t^2)\right)-\\\alpha(t-\gamma)^{\beta}\left((t_4-t)+\left(\frac{\delta}{2}(t_4^2-t^2)\right)\right)\end{array}\right\}\right]dt\right)\end{array}\right]$$

For simplifying above equation, it is expanded along exponential function. For remove the complexity of above equation and making calculation easier, higher power of α will be avoided.

$$\text{DC}=\frac{P_c}{T}\left[\begin{array}{l}\left(\left[(P-D(p))\left\{\alpha\left(t_1(t_1-\gamma)^{\beta}-\frac{(t_1-\gamma)^{\beta+1}}{\beta+1}+\frac{(-\gamma)^{\beta+1}}{\beta+1}\right)\right\}\right]\right)\\+\left[a(P-D(p))\left\{\alpha\left(t_2(t_2-\gamma)^{\beta}-t_1(t_1-\gamma)^{\beta}-\left(\frac{(t_2-\gamma)^{\beta+1}-(t_1-\gamma)^{\beta+1}}{(\beta+1)}\right)\right)\right\}\right]\\+\left[b(P-D(p))\left\{\alpha\left(t_3(t_3-\gamma)^{\beta}-t_2(t_2-\gamma)^{\beta}-\left(\frac{(t_3-\gamma)^{\beta+1}-(t_2-\gamma)^{\beta+1}}{(\beta+1)}\right)\right)\right\}\right]\\+D(p)\left((t_3-\gamma)^{\beta}(t_3-t_4)+\alpha\left(\frac{(t_4-\gamma)^{\beta+1}-(t_3-\gamma)^{\beta+1}}{(\beta+1)}\right)\right)\end{array}\right] \tag{24}$$

Shortage Cost/unit time

$$SC=\frac{S_c}{T}\left[\left(\int_{t_4}^{t_5}Q(t)dt+\int_{t_5}^{T}Q(t)dt\right)\right]$$

$$=\frac{S_c}{T}\left(\begin{array}{l}D(p)\left(-\frac{(t_5-t_4)^2}{2}+\frac{\delta}{6}(t_5-t_4)^3\right)+\\-\left((P-D(p))\left(\frac{1}{2}(T-t_5)^2+\frac{\delta}{2}\left(\frac{2T^3}{3}-T^2t_5+\frac{t_5^3}{3}\right)+\delta\left(\frac{T^3}{6}-\frac{Tt_5^2}{2}+\frac{t_5^3}{3}\right)\right)\right)\end{array}\right)$$

Now by keeping the value of t5 in above equation, we obtained

$\{\frac{1}{T}S_c(D(-\frac{1}{2}(-k_3t_1+1/(P\delta)\,0.16666666666666666(2.D-4.P-2.DT\delta+2.PT\delta+2.D\delta k_3t_1-\sqrt{((-2.D+4.P+2.DT\delta-2.PT\delta-2.D\delta k_3t_1)^2-12.P\delta(4.DT-4.PT+DT^2\delta-1.PT^2\delta-2.Dk_3t_1-1.D\delta k_3^2t_1^2)))})^2+\frac{1}{6}\delta(-k_3t_1+1/(P\delta)\,0.16666666666666666(2.D-4.P-2.DT\delta+2.PT\delta+$

$2.D\delta k_3 t_1 - \sqrt{}((-2.D + 4.P + 2.DT\delta - 2.PT\delta - 2.D\delta k_3 t_1)^2 - 12.P\delta(4.DT - 4.PT + DT^2\delta - 1.PT^2\delta - 2.Dk_3 t_1 - 1.D\delta k_3^2 t_1^2))))^3) - (-D + P)(\frac{1}{2}(T - 1/(P\delta)\, 0.16666666666666666(2.D - 4.P - 2.DT\delta + 2.PT\delta + 2.D\delta k_3 t_1 - \sqrt{}((-2.D + 4.P + 2.DT\delta - 2.PT\delta - 2.D\delta k_3 t_1)^2 - 12.P\delta(4.DT - 4.PT + DT^2\delta - 1.PT^2\delta - 2.Dk_3 t_1 - 1.D\delta k_3^2 t_1^2))))^2 + \frac{1}{2}\delta(\frac{2T^3}{3} - 1/(P\delta)\, 0.16666666666666666T^2(2.D - 4.P - 2.DT\delta + 2.PT\delta + 2.D\delta k_3 t_1 - \sqrt{}((-2.D + 4.P + 2.DT\delta - 2.PT\delta - 2.D\delta k_3 t_1)^2 - 12.P\delta(4.DT - 4.PT + DT^2\delta - 1.PT^2\delta - 2.Dk_3 t_1 - 1.D\delta k_3^2 t_1^2))) + 1/(P^3\delta^3)\, 0.0015432098765432094(2.D - 4.P - 2.DT\delta + 2.PT\delta + 2.D\delta k_3 t_1 - \sqrt{}((-2.D + 4.P + 2.DT\delta - 2.PT\delta - 2.D\delta k_3 t_1)^2 - 12.P\delta(4.DT - 4.PT + DT^2\delta - 1.PT^2\delta - 2.Dk_3 t_1 - 1.D\delta k_3^2 t_1^2)))^3) + \delta(\frac{T^3}{6} - 1/(P^2\delta^2)\, 0.013888888888888888T(2.D - 4.P - 2.DT\delta + 2.PT\delta + 2.D\delta k_3 t_1 - \sqrt{}((-2.D + 4.P + 2.DT\delta - 2.PT\delta - 2.D\delta k_3 t_1)^2 - 12.P\delta(4.DT - 4.PT + DT^2\delta - 1.PT^2\delta - 2.Dk_3 t_1 - 1.D\delta k_3^2 t_1^2)))^2 + 1/(P^3\delta^3)\, 0.0015432098765432094(2.D - 4.P - 2.DT\delta + 2.PT\delta + 2.D\delta k_3 t_1 - \sqrt{}((-2.D + 4.P + 2.DT\delta - 2.PT\delta - 2.D\delta k_3 t_1)^2 - 12.P\delta(4.DT - 4.PT + DT^2\delta - 1.PT^2\delta - 2.Dk_3 t_1 - 1.D\delta k_3^2 t_1^2)))^3))))\}]$ (25)

Emission cost

In a production supply chain carbon emission is due to production process If the nature of inventory is deteriorating type, then it will definitely leads to carbon emission and according to Government policy any business organization that is responsible for carbon emission, have to pay to government as a penalty following by any one policy established by Government. In our work we have considered carbon tax policy. For this let's assume that carbon generation by deteriorating inventories is $\nu\, Kg/unit$.

Therefore, total emission due to disposal of deteriorating inventories

$$= \frac{\nu}{T}\left[\int_0^{t_1}\phi(t)Q(t)dt + \int_{t_1}^{t_2}\phi(t)Q(t)dt + \int_{t_2}^{t_3}\phi(t)Q(t)dt + \int_{t_3}^{t_4}\phi(t)Q(t)dt\right]$$

Now let,s assume that for each kg per unit of carbon emission generation the supply chain bears a tax of ₹ $\chi/kg/unit$. Therefore, total green impact cost incurred is given by

$$= \frac{\nu\chi}{T}\left[\int_0^{t_1}\phi(t)Q(t)dt + \int_{t_1}^{t_2}\phi(t)Q(t)dt + \int_{t_2}^{t_3}\phi(t)Q(t)dt + \int_{t_3}^{t_4}\phi(t)Q(t)dt\right]$$

$$\frac{\chi\nu}{T}\left[\begin{array}{l}\left(\int_0^{t_1}\alpha\beta(t-\gamma)^{\beta-1}\left[(P-D(p))\left\{t+\frac{\delta t^2}{2}+\frac{\alpha}{\beta+1}\left((t-\gamma)^{\beta+1}-(-\gamma)^{\beta+1}\right)-\left(t+\frac{\delta t^2}{2}\right)\alpha(t-\gamma)^{\beta}\right\}\right]dt\right)\\ +\left(\int_{t_1}^{t_2}\alpha\beta(t-\gamma)^{\beta-1}\left[a(P-D(p))\left\{t+\frac{\delta t^2}{2}+\frac{\alpha}{\beta+1}\left((t-\gamma)^{\beta+1}-(-\gamma)^{\beta+1}\right)-\left(t+\frac{\delta t^2}{2}\right)\alpha(t-\gamma)^{\beta}\right\}\right]dt\right)\\ +\left(\int_{t_2}^{t_3}\alpha\beta(t-\gamma)^{\beta-1}\left[b(P-D(p))\left\{t+\frac{\delta t^2}{2}+\frac{\alpha}{\beta+1}\left((t-\gamma)^{\beta+1}-(-\gamma)^{\beta+1}\right)-\left(t+\frac{\delta t^2}{2}\right)\alpha(t-\gamma)^{\beta}\right\}\right]dt\right)\\ +\left(\int_{t_3}^{t_4}\alpha\beta(t-\gamma)^{\beta-1}\left[D(p)\left\{\begin{array}{l}(t_4-t)+\frac{\alpha}{\beta+1}\left((t_4-\gamma)^{\beta+1}-(t-\gamma)^{\beta+1}\right)+\left(\frac{\delta}{2}(t_4^2-t^2)\right)-\\ \alpha(t-\gamma)^{\beta}\left((t_4-t)+\left(\frac{\delta}{2}(t_4^2-t^2)\right)\right)\end{array}\right\}\right]dt\right)\end{array}\right]$$

$$\frac{\chi v}{T}\left[\begin{array}{l}\left(\left[(P-D(p))\left\{\alpha\left(t_1(t_1-\gamma)^{\beta}-\frac{(t_1-\gamma)^{\beta+1}}{\beta+1}+\frac{(-\gamma)^{\beta+1}}{\beta+1}\right)\right\}\right]\right)\\ +\left[a(P-D(p))\left\{\alpha\left(t_2(t_2-\gamma)^{\beta}-t_1(t_1-\gamma)^{\beta}-\left(\frac{(t_2-\gamma)^{\beta+1}-(t_1-\gamma)^{\beta+1}}{(\beta+1)}\right)\right)\right\}\right]\\ +\left[b(P-D(p))\left\{\alpha\left(t_3(t_3-\gamma)^{\beta}-t_2(t_2-\gamma)^{\beta}-\left(\frac{(t_3-\gamma)^{\beta+1}-(t_2-\gamma)^{\beta+1}}{(\beta+1)}\right)\right)\right\}\right]\\ +D(p)\left((t_3-\gamma)^{\beta}(t_3-t_4)+\alpha\left(\frac{(t_4-\gamma)^{\beta+1}-(t_3-\gamma)^{\beta+1}}{(\beta+1)}\right)\right)\end{array}\right] \tag{26}$$

Therefore , it is the total tax that supply chain needs to pay for per kg per unit carbon generation.

Total earn revenue

If selling price be p then total revenue earn is

$$RC=\frac{p}{T}\left[\int_0^{t_1}D((p),Q(t))dt+\int_{t_1}^{t_2}D((p),Q(t))dt+\int_{t_2}^{t_3}D((p),Q(t))dt+\int_{t_3}^{t_4}D((p),Q(t))dt\right]$$

$$\frac{p}{T}\left[\begin{array}{l}\left(\int_0^{t_1}\left(D(p)+\delta(P-D)\left\{\begin{array}{l}t+\frac{\delta t^2}{2}+\frac{\alpha}{\beta+1}\left((t-\gamma)^{\beta+1}-(-\gamma)^{\beta+1}\right)\\ -\left(t+\frac{\delta t^2}{2}\right)\alpha(t-\gamma)^{\beta}\end{array}\right\}\right)dt\right)+\\ \left(\int_{t_1}^{t_2}\left(D(p)+\delta a(P-D)\left\{\begin{array}{l}t+\frac{\delta t^2}{2}+\frac{\alpha}{\beta+1}\left((t-\gamma)^{\beta+1}-(-\gamma)^{\beta+1}\right)\\ -\left(t+\frac{\delta t^2}{2}\right)\alpha(t-\gamma)^{\beta}\end{array}\right\}\right)dt\right)\\ \left(\int_{t_2}^{t_3}\left(D(p)+\delta b(P-D)\left\{\begin{array}{l}t+\frac{\delta t^2}{2}+\frac{\alpha}{\beta+1}\left((t-\gamma)^{\beta+1}-(-\gamma)^{\beta+1}\right)\\ -\left(t+\frac{\delta t^2}{2}\right)\alpha(t-\gamma)^{\beta}\end{array}\right\}\right)dt\right)\\ \left(\int_{t_3}^{t_4}\left(D(p)+\delta D(p)\left\{\begin{array}{l}(t_4-t)+\frac{\alpha}{\beta+1}\left((t_4-\gamma)^{\beta+1}-(t-\gamma)^{\beta+1}\right)+\left(\frac{\delta}{2}(t_4^2-t^2)\right)-\\ \alpha(t-\gamma)^{\beta}\left((t_4-t)+\left(\frac{\delta}{2}(t_4^2-t^2)\right)\right)\end{array}\right\}\right)dt\right)\end{array}\right]$$

$$\frac{p}{T}\begin{bmatrix} \left(D(p)t_1+\delta(P-D)t_1\right)+ \\ \left(D(p)\left(t_2-t_1\right)+\delta a(P-D)\left(t_2-t_1\right)\right) \\ \left(D(p)\left(t_3-t_2\right)+\delta b(P-D)\left(t_3-t_2\right)\right) \\ \left(D(p)\left(t_4-t_3\right)+\delta D(p)\frac{\left(t_4-t_3\right)^2}{2}\right) \end{bmatrix} \quad (27)$$

After calculating the above occurring cost and revenue for one cycle for manufacturing supply chain, total profit function is given by

$$TP^* = \left(RC-PC-SP-HC-DC-SC-EC\right) \quad (28)$$

The problem can be stated as an optimization problem and it can be formulated as

By using equation (21), (22), (23), (24), (25), (26), (27) and $t_2=k_1t_1$, $t_3=k_2t_1$ $t_4=k_3t_1$ and $\beta=2$ in equation (28), Total profit model for discussed problem will be obtained.

Maximize [$TP(t_1,T,v,p)$]

Subject to $0\le t_1, 0\le T, 0<p\le\frac{x_1}{y}, v>0$

8. NUMERICAL ANALYSIS

$$H=5,\tau=0.4,x_1=400,x_2=90,y=80,\delta=0.40,\chi=Rs\,4/kg/unit,\beta=2,P_c=50/unit,P=1000\,units,$$
$$S_p=60/unit\ time,\mu=1.01,S_c=4/unit,a=0.18,b=0.15,\alpha=0.001,\gamma=0.40,k_1=1.01,k_2=0.76,k_3=0.78$$

Optimum values of decision variables that are calculated on the bases of above numerical example

$$p=0.00330056088/unit,T=0.976568,v=3.4685390kg/unit,t_1=0.861804275$$

Optimal value of profit $TP^*=1.0871946\times10^9$

THEOREM 1- There exist a point of maxima for profit function with respect to p and T.

Proof- To prove the existence of maximum point for profit function with respect to T and p, we use the method of hessian matrix. It is determined that whether the hessian matrix is positive or negative definite. There exist a maximum point if and only if hessian matrix is negative definite.

$$H(T,p)=\begin{bmatrix} \frac{\partial^2 TP^*}{\partial T^2} & \frac{\partial^2 TP^*}{\partial p\partial T} \\ \frac{\partial^2 TP^*}{\partial p\partial T} & \frac{\partial^2 TP^*}{\partial p^2} \end{bmatrix}$$

After solving, we get $\frac{\partial^2 TP^*}{\partial T^2}>0$, $\frac{\partial^2 TP^*}{\partial p^2}<0$, $\frac{\partial^2 TP^*}{\partial T\partial p}>0$ and $\left|H(T,p)\right|<0$, that proves the theorem.

THEOREM 2- There exist a point of maxima for profit function with respect to T and t_1.

Proof- To prove the existence of maximum point for profit function with respect to T and t_1, we use the method of hessian matrix. It is determined that whether the hessian matrix is positive or negative definite. There exist a maximum point if and only if hessian matrix is negative definite.

$$H(T,t_1)=\begin{bmatrix}\frac{\partial^2 TP^*}{\partial T^2} & \frac{\partial^2 TP^*}{\partial t_1 \partial T}\\ \frac{\partial^2 TP^*}{\partial t_1 \partial T} & \frac{\partial^2 TP^*}{\partial t_1{}^2}\end{bmatrix}$$

After solving, we get $\frac{\partial^2 TP^*}{\partial T^2}>0$, $\frac{\partial^2 TP^*}{\partial t_1{}^2}<0$, $\frac{\partial^2 TP^*}{\partial T \partial t_1}>0$ and , $|H(T,t_1)|<0$, that proves the theorem.

THEOREM 3- There exist a point of maxima for profit function with respect to p and t_1.

Proof- To prove the existence of maximum point for profit function with respect to p and t_1, we use the method of hessian matrix. It is determined that whether the hessian matrix is positive or negative definite. There exist a maximum point if and only if hessian matrix is negative definite.

$$H(p,t_1)=\begin{bmatrix}\frac{\partial^2 TP^*}{\partial p^2} & \frac{\partial^2 TP^*}{\partial p \partial t_1}\\ \frac{\partial^2 TP^*}{\partial t_1 \partial p} & \frac{\partial^2 TP^*}{\partial t_1{}^2}\end{bmatrix}$$

After solving, we get $\frac{\partial^2 TP^*}{\partial p^2}>0$, $\frac{\partial^2 TP^*}{\partial t_1{}^2}>0$, $\frac{\partial^2 TP^*}{\partial p \partial t_1}<0$ and $|H(p,t_1)|<0$, that proves the theorem.

THEOREM 4-There exist a point of maxima for profit function with respect to p and ν.

Proof- To prove the existence of maximum point for profit function with respect to p and ν, we use the method of hessian matrix. It is determined that whether the hessian matrix is positive or negative definite. There exist a maximum point if and only if hessian matrix is negative definite.

$$H(p,\nu)=\begin{bmatrix}\frac{\partial^2 TP^*}{\partial p^2} & \frac{\partial^2 TP^*}{\partial p \partial \nu}\\ \frac{\partial^2 TP^*}{\partial \nu \partial p} & \frac{\partial^2 TP^*}{\partial \nu^2}\end{bmatrix}$$

After solving, we get $\frac{\partial^2 TP^*}{\partial p^2}>0$, $\frac{\partial^2 TP^*}{\partial \nu^2}=0$, $\frac{\partial^2 TP^*}{\partial p \partial \nu}>0$ and $|H(p,\nu)|<0$, that proves the theorem.

Table 1 - Sensitivity analysis of profit function with respect to decision variables

t_1	T	p	ν	TP^*
0.8618042	0.9765680	0.00330056	3.46853904	1.086764521×10^9
0.8718042	0.9765680	0.00330056	3.46853904	1.1129662×10^9
0.8818042	0.9765680	0.00330056	3.46853904	1.1394763×10^9
0.8918042	0.9765680	0.00330056	3.46853904	1.1662947×10^9
0.8618042	0.9865680	0.00330056	3.46853904	1.0785598×10^9
0.8618042	0.9895680	0.00330056	3.46853904	1.0761253×10^9
0.8618042	0.9995680	0.00330056	3.46853904	1.068098×10^9
0.8618042	0.9765680	0.00334056	3.46853904	1.065713×10^9
0.8618042	0.9765680	0.00336056	3.46853904	1.055388×10^9
0.8618042	0.9765680	0.00338056	3.46853904	1.0451949×10^9

0.8618042	0.9765680	0.00330056	3.47853904	1.086764521×10^9
0.8618042	0.9765680	0.00330056	3.48853904	1.086764520×10^9
0.8618042	0.9765680	0.00330056	3.59853904	1.0867645173×10^9
0.8618042	0.9765680	0.00330056	3.69853904	1.0867645142×10^9

Table 2 - Changes in values of profit TP^* with respect to different parameters

Parameters	Change	TP^*	Parameters	Change	TP^*
μ	1.01	1.08676×10^9	a	0.19	1.184930×10^9
	1.02	1.29970×10^9		0.20	1.17946×10^9
	1.03	1.413585×10^9		0.21	1.17400×10^9
τ	0.41	1.15971×10^9	b	0.6	1.1903904×10^9
	0.42	1.12902×10^9		0.7	1.1903902×10^9
	0.43	1.09836×10^9		0.8	1.1903901×10^9
x_1	410	1.19082×10^9	k_1	0.32	2.540008×10^9
	420	1.191252×10^9		0.34	3.88962×10^9
	430	1.1916831×10^9		0.36	5.239243×10^9
x_2	92	1.231818×10^9	k_2	0.77	1.1903898×10^9
	94	1.27312×10^9		0.78	1.1903891×10^9
	96	1.3142×10^9		0.79	1.190388×10^9
y	82	1.19039027×10^9	k_3	0.76	1.187470×10^9
	83	1.19039012×10^9		0.77	1.18452×10^9
	84	1.1903899×10^9		0.78	1.18155×10^9
δ	0.41	1.17841×10^9	α	0.002	1.19039067×10^9
	0.42	1.165660×10^9		0.003	1.190390797×10^9
	0.43	1.1521×10^9		0.005	1.19039103×10^9
γ	0.42	1.19039063×10^9	β	3	1.1903909×10^9
	0.44	1.19039071×10^9		4	1.19039106×10^9
	0.46	1.19039077×10^9		5	1.190391079×10^9

9. SENSITIVITY ANALYSIS

To check the applicability of designed model for a manufacturing supply chain sensitivity analysis carry out as to behaviour of profit function with respect to a, b, k_1, k_2, k_3, μ, τ, x_1, x_2, y γ and δ can be analyzed.

1. In table1, it is presented how profit function TP^* changes with respect to decision variables t_1, T, p, v. As the price p, the carbon emission factor v and cycle length T increases , total profit TP^* is decreases that gives an inverse relation between price p, cycle length T and carbon emission factor v and as the production period t_1 increases profit is increases. Graphical representations also given of profit TP^* with respect to price, emission factor and cycle length and production period.

2. In table 2, behaviour of profit with respect to different model's parameters have been given. As the values of parameters k_1, μ, γ, x_1, x_2, α and β increasing value of profit function TP^* is increasing but as values of k_2, k_3, τ, a, b, δ and y increasing value of profit function TP^* is decreasing. This shows an inverse relation between profit function TP^*

and the parameters k_2 , k_3, τ , a , b , δ and y and direct relation between profit and the parameters k_1 , μ , γ , x_1 , x_2 , α and β .

3.Graphical representation of relation of profit with respect to different decision variables and other parameters also provided from figure 3 to figure 12.

4. Figure 4 represent if price increases demand is decreases that is true as per practical point of view as well as it supporting our assumption also.

5. Figure 3 shows inverse relation between price and profit and proved the concept that excess increment in the price may cause of loss. As it is shown in figure 4 and in table 1that as price increase demand decrease and it leads to decrease in profit.

6. Figure 5 shows behaviour of profit with respect to α . As value of α increases profit is also increases as α means range of decaying period so if range will increases ,profit is increases because it increase the time of deterioration. In figure 6 profit graph going upward as t_1 increases that shows direct relation between t_1 and profit. Same way figure 7 and figure 8 representing inverse behaviour of profit with respect to v and τ .As value of v and τ increasing profit is decreasing.

7. Figure 9, figure 10, figure 11 and figure 12 representing behaviour of profit with respect to decision variables T and t_1 , t_1 and p , p and T and T and v respectively which shows how profit graphs changes as increase or decrease in decision variables. Profit increases with increase in t_1 and decreases with increase in p , T and v .

10. OBSERVATION

By sensitivity analysis, following observations has been made.

1. By table1, it is observed that with increase in price p , the carbon emission factor v and cycle length T , total profit TP^* is decreases that implies an inverse relation between these decision variables and profit. To get higher profit we need to minimize price p , cycle length T and carbon emission factor v . It is also observed by table 1 that as the production period t_1 increases profit is increases, so for higher profit production period t_1 ,we need to maximize t_1 but it should be smaller than cycle length T . It is observed that if the price is less, it increases the demand and inventory would be selling out before expiry date that leads to high profit. Graphical representations also provided to justify the result. As the values of T increases profit is decreases and as the values of t_1 increases profit is increases. Therefore large cycle length and high price both x_1 are reasons of loss in a production supply chain. Small cycle length is easy to manage also. If the production period is short and total cycle length is much larger as compare to production period the rate of deterioration will be increases but if production period is long and total cycle length is not much longer as compare to production period rate of deterioration will be decrease and it leads to higher profit. If we consider carbon emission factor, by table 1 it is observed that as carbon emission factor increases profit also increases.

2. By table 2, behaviour of model parameters have been seen and it is observed that as the values of parameters k_1 , μ , γ , x_1 , x_2 , α and β increasing value of profit function TP^* is increasing but as values of k_2 , k_3 , τ , a , b , δ and y increasing value of profit function TP^* is decreasing. This shows an inverse relation between profit function TP^* and the parameters k_2 , k_3 , τ , a , b , δ and y which implies that to increase profit of supply chain, these parameter needs to be decrease. Same way profit is directly related to parameters k_1 , μ , γ , x_1 , x_2 , α and β which implies that to increase profit of supply chain, these parameter needs to be increase.

3. In graphical representation in figure 3 and 4 relation of profit with respect to demand and price can be observed that if price increases demand is decreases and as a result profit is increases. It is true as per practical point of view as well as it supporting our assumption also. We know that increase in price decreases market demand that leads to decrease in profit therefore the concept that excess increment in the price may cause of loss.

4. It is observed in figure 5 that as the value of α increases profit is also increase because α is here range of deterioration and if range will be larger then deterioration starts after more time that leads to profit. Figure 6 showing direct relation between t_1 and profit, it is observed that production period should be larger for increment in profit. Same way figure 7 and figure 8 representing inverse behaviour of profit with respect to ν and τ .As value of τ increasing profit is decreasing and as carbon emission increase profit decreases. For higher profit we need to control carbon emission factor.

5. Figure 9 to figure 12 representing relation between decision variables and profit. It is observed by all mathematical calculation and graphs that profit increases if production period t_1 is longer and decreases with increase in values of cycle length T , price p and carbon emission factor ν .Therefore for higher profit we need to minimizes cycle length T , price p and carbon emission factor ν and need to maximize production period with this condition that production period should be less that total cycle length.

11. CONCLUSION

It is difficult to meet profit targets in any business that deals with decaying items since product life is not predictable and varies on a variety of circumstances. The cheap pricing of items is a key component that may assist to enhance profit. Lowering the price boosts demand and causes goods to sell out before the expiry date, hence lowering the price may enhance profit. Another factor is that if the production period is short and the total cycle length is much longer than the production period, the rate of deterioration increases, resulting in supply chain loss, whereas if the production period is long and the total cycle length is much shorter, the rate of deterioration decreases, resulting in supply chain profit. In this paper, both notions are investigated and justified. Along with this, it is being researched if increasing the carbon emission factor leads to further loss because increasing the carbon emission factor leads to greater taxes. As a result, carbon emissions must be reduced for both economic and social reasons. The current work is a manufacturing supply chain with tiny production sub-cycles within a single cycle. It is regarded as price and stock dependent demand, as well as the Weibull degradation rate. In our research, we used a three-parameter Weibull distribution to determine the decay rate. For goods whose rate of degradation rises with time, the Weibull distribution is more practical and versatile. Mathematical modelling with the Weibull distribution for rate of decay provides greater model flexibility since it may be used with any sort of rate of decay. Many commodities begin the degradation process right once, while certain products have a shelf life. As a result, we adopted the Weibull distribution so that this model may be applied for any decaying natural inventory. The goal of this project is to maximize profit. We demonstrated all of the aforementioned arguments in our conclusion using mathematical modelling and sensitivity analysis. Based on the numerical study of the model parameter, it is found that for the Above manufacturing supply chain cycle length should be reduced, production period increased, and price and carbon emissions reduced. It is also determined that when prices rise, demand falls and, as a result, profit falls. It is also established that the longer the production duration, the higher the profit. This work is applicable for the manufacturing firms dealing with deteriorating nature inventory where production process takes place in small sub cycles. This study may be expanded with alternative carbon emission control policies, deterioration rates, and demand rates.

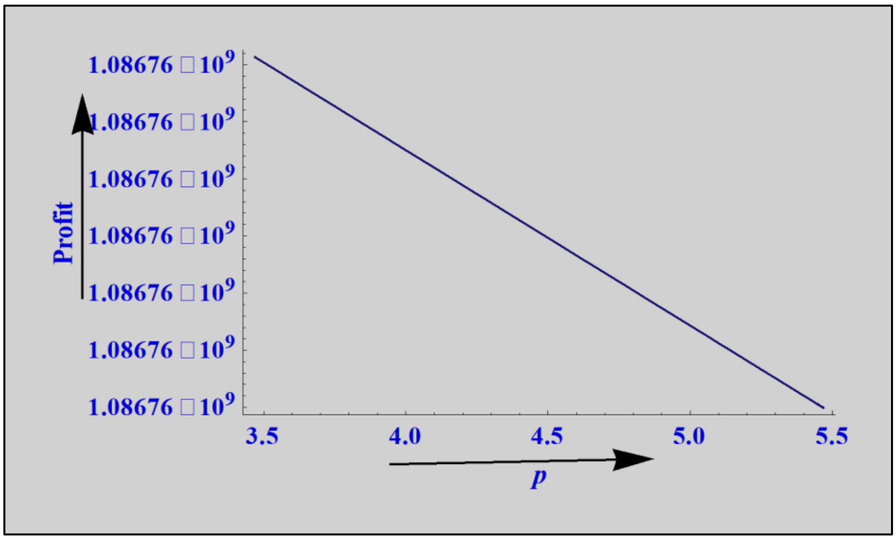

Figure 3 - Profit with respect to price

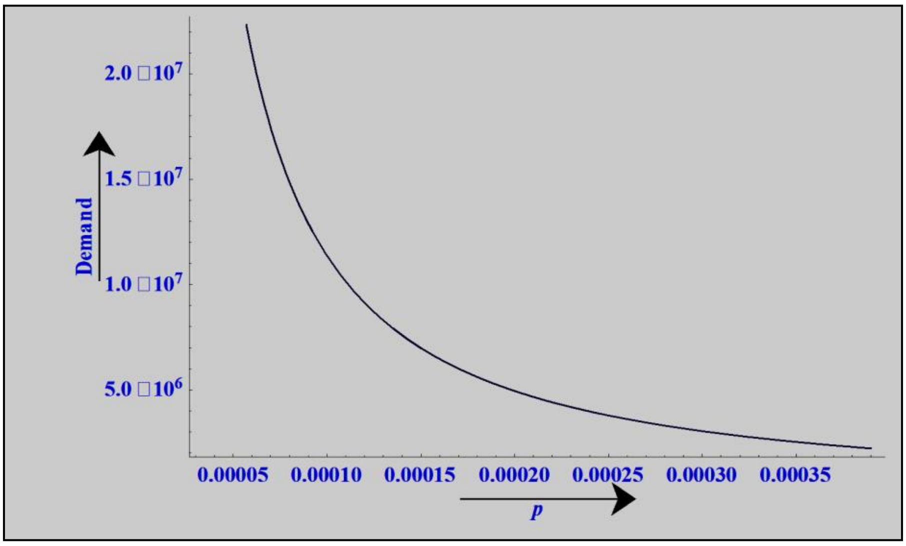

Figure 4 - Relation between demand and price

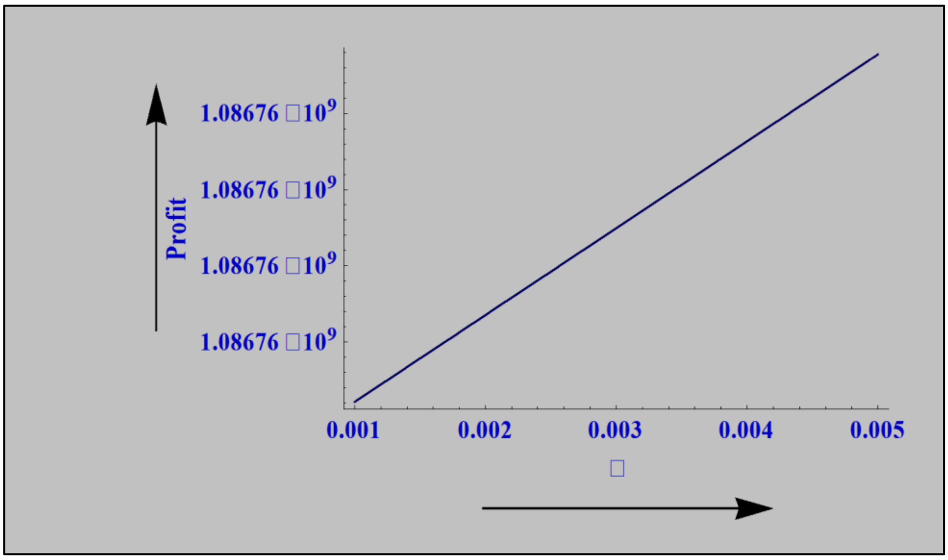

Figure 5 - Profit with respect to α

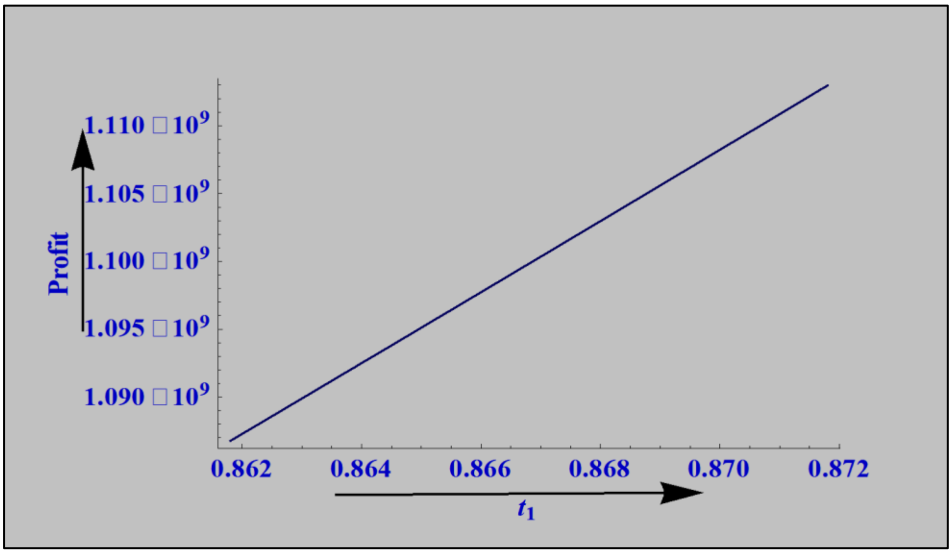

Figure 6 - Profit with respect to t_1

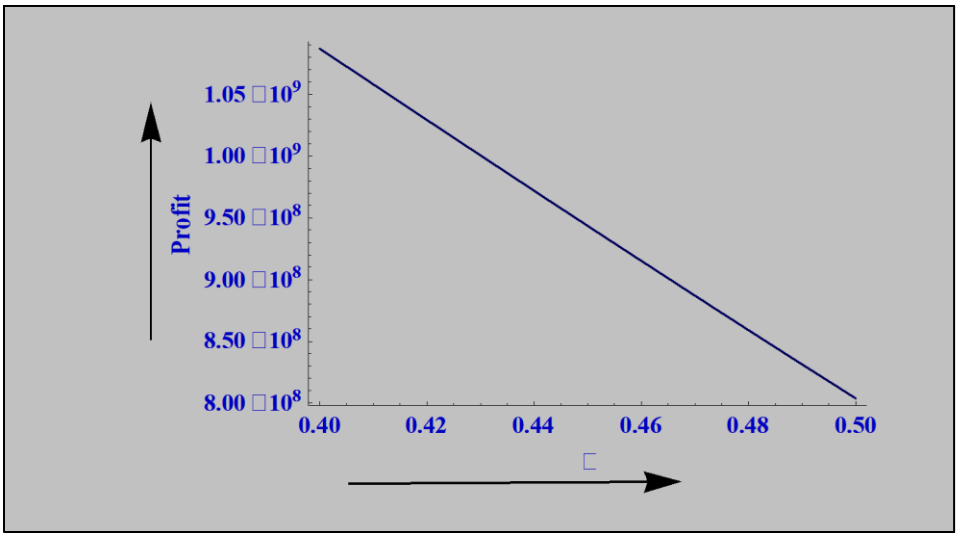

Figure 7 - Profit with respect to τ

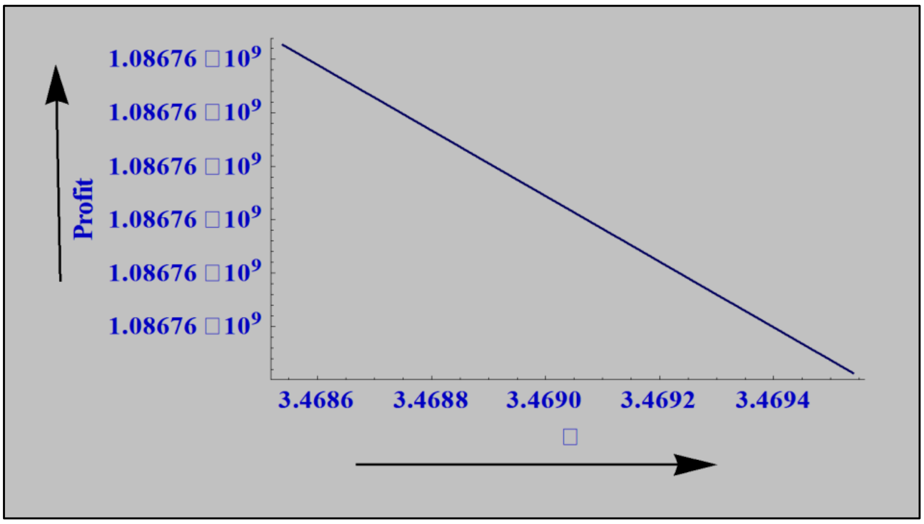

Figure 8 - Profit with respect to V

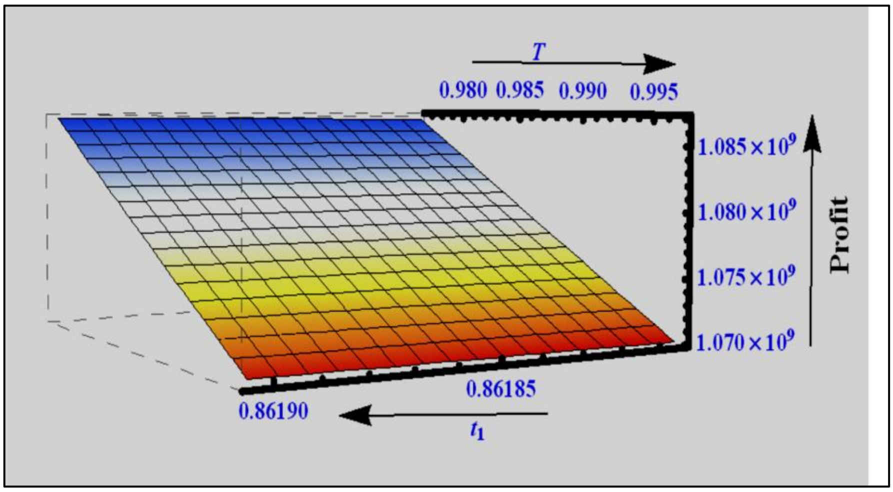

Figure 9 - Profit with respect to t_1 and T

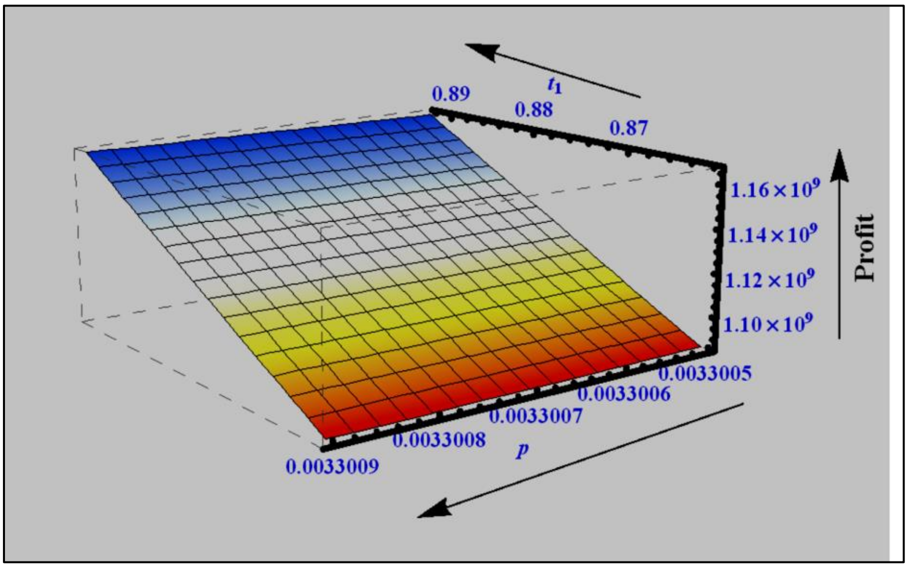

Figure 10 -Profit with respect to t_1 and p

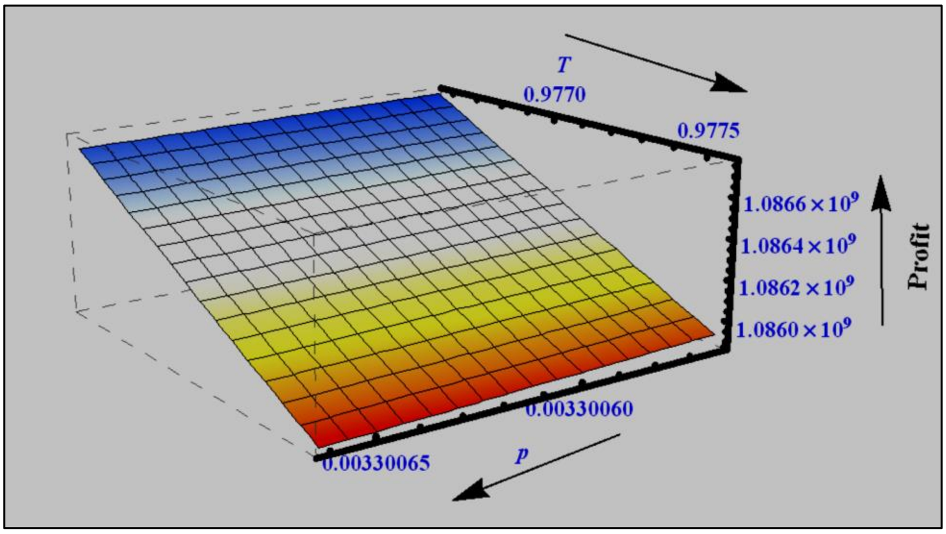

Figure 11 -Profit with respect to T and p

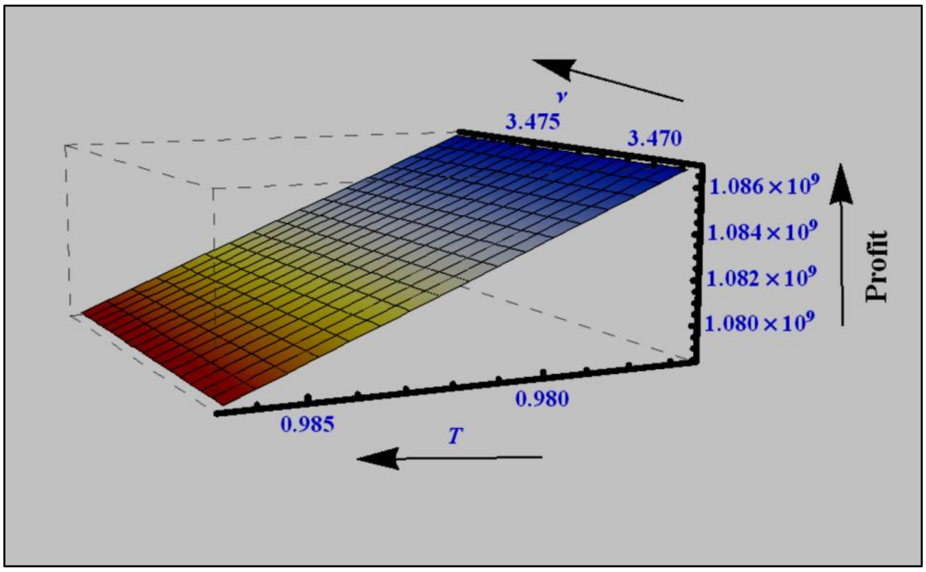

Figure 12 -Profit with respect to T and v

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